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The causes of changes in student positioning in group discussions using Polya's problem-solving and *commognitive* approaches

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Abstract

This study aims to describe the causes of changes in student positioning during group discussions utilizing Polya's problem-solving method and commognitive principles. The subjects of the study are mathematics education students from STKIP Taman Siswa Bima. Two groups were selected from 30 participants, each consisting of three individuals representing the roles of expert, facilitator, and novice. The research approach employed was qualitative exploratory. The findings indicate that changes in positioning were due to role shifts among group members, where group 1 changed from an initial positioning of EP1FP1NP1 to F'P1E'P1NP1, incorporating complete stages of Polya's problem-solving process and commognitive components. Group 2 transitioned from EP2FP2NP2 to F'P2FP2NP2, reflecting an incomplete application of Polya's problem-solving stages and commognitive components. Future research recommendations include examining how changes in student positioning occur in algebra problem-solving discussions using Polya's problem-solving strategies and commognitive components, with a focus on individual student problem-solving within groups.

Keywords: positioning, group discussion, Polya's problem-solving, commognitive, algebra

INTRODUCTION

Group discussions are a crucial component of the learning activities found in nearly every instructional model or strategy in the classroom. Given the importance of group discussions in mathematics education policies and curriculum content, it is essential to understand how students comprehend mathematics during these discussions (DeJarnette & González, 2015). Similarly, college-level mathematics curricula emphasize problem-solving that involves group discussions. In line with this, group discussions teach students to learn how to listen, respond to their peers' ideas, construct arguments, and engage in problem-solving efforts (Deal & Wismer, 2010; Kavanagh et al., 2020). Since group discussions offer numerous benefits in mathematics learning, it is important to examine how students participate in group discussions during mathematics instruction.

Participation or involvement of members in group discussions can maximize individual contributions, making it easier to achieve common goals (Johnson et al., 2014; Nilsson & Ryve, 2010). Group discussions require collaboration, where students are capable of developing a shared understanding of the mathematics content they master, leading to the generation of collective ideas (Staples, 2014). Retnowati et al. (2016) also assert that the goal of group discussions in mathematics education is for students to actively engage in collaborative learning. Furthermore, another supportive activity during group discussions is observing others' strategies, allowing students to imitate or adopt ideas from peers they find effective as well as having the opportunity to express their own ideas (Yu & Hu, 2017). Based on these perspectives, the overall activities during group discussions can lead to positive mathematics learning, particularly in how students can develop interaction and communication skills that foster collaborative learning.

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Contribution to the literature

- This study addresses the limitations in research regarding changes in group discussion positioning in mathematics education.
- The research identifies the causes of changes in group discussion positioning through the use of Polya's problem-solving strategy and *commognitive* approach.
- This study also illustrates the impact of changes in group discussion positioning utilizing Polya's strategy and commognitive principles as solutions for algebra problem-solving.

In student interactions, ideas are formed through both reflection and the exchange of opinions during discussions (Cobb et al., 2012). When students encourage their peers to revisit and refine their mathematical knowledge over time, their conceptual understanding can develop (Francisco, 2013). For instance, in some cases, students may collaborate to construct an idea, while in other instances, an individual student may present their idea comprehensively for further development or modification by others (Mueller et al., 2012). However, students may not necessarily be skilled at communicating their ideas in mathematics classes (Cole, 2010). This is due to the fact that communication in mathematics differs from the narratives used in daily life, which necessitates effective interaction in group discussions. Although differences of opinion may arise during group activities, students can still benefit from these experiences (Forman et al., 2015). When students establish patterns in their interactions during group discussions, such interactions enable them to express their mathematical ideas. As Lee and Martin (2017) indicate, group discussions expand learning opportunities for students, as they not only focus on finding the correct answers but also engage in exploring their existing knowledge.

During the interaction process, it is essential to consider how students position themselves in relation to one another during group work (Esmonde, 2013). In a group setting, a student's identity can shift based on how they are positioned by their peers and the instructor (Wood, 2013). In some groups, a student may take on the role of a teacher, instructing and evaluating their classmates (Tholander & Aronsson, 2014). Throughout mathematics discussions, students continuously renegotiate their positions concerning their peers (DeJarnette, 2018). Based on expert opinions, it is clear that positioning is crucial in group discussion activities.

The theory of positioning in group discussions explains how students interact while collaborating. Positioning arises from a narrative thread that weaves together events with the content of the story (Wood, 2013). For example, it includes how an instructor positions themselves while standing at the front of the classroom, as well as how students position themselves when paying attention to the instructor's guidance. Students are often positioned under the direction and authority of the instructor (Herbel-Eisenmann et al., 2015; Struyve et al., 2018). Positioning refers to the manner in which individuals place themselves and communicate during interactions (DeJarnette & González, 2015). Thus, in general, positioning determines how one lays out their stance in relation to others.

The theory of positioning is a highly valuable construct as it provides a lens to examine how students negotiate their rights and responsibilities for participating in small groups (van Langenhove & Harré, 1999). Positioning is an approach to analyzing patterns of interpersonal actions created by individuals involved in the disclosure of social episodes, where rights and responsibilities are created and maintained through social interactions among the participants present in those episodes (Harré, 2015). Furthermore Kayı-Aydar (2019) reveals that positioning studies the rights, duties, and responsibilities distributed among interlocutors or characters through conversation or narrative, aiming to understand how these rights and responsibilities shape the social structure as they are formed. The rights and responsibilities in this positioning occur within the dynamic context of group discussions (Alvermann et al., 2018). There are three types of dynamic positions that each participant can assume in group discussions: expert, novice, and facilitator (Kayı-Aydar, 2019).

Research on positioning related to group discussions has been conducted by several previous researchers at the secondary school level, including Daher (2020) and Esmonde (2013) as well as at the college level, such as DeJarnette and González (2015) and Zhang et al. (2019), and encompassing both schools and higher education (Campbell & Hodges, 2020). Esmonde (2013) investigated student group interactions using positioning theory to compare two different activities: group quizzes and group presentations on the topic of functions. The results indicated that groups without an expert tended to have passive discussions and lacked dynamic interactions. In groups that included both experts and novices, without a facilitator, discussions tended to be dominated by the expert, limiting opportunities for the novice to explain. Collaborative groups, however, tend to be much fairer, as all members have the opportunity to contribute to collective problem-solving. In this context, the research has not specifically examined changes in positioning during group discussions.

The research conducted by DeJarnette and González (2015) utilized positioning theory to examine the patterns of positioning demonstrated by students during group work and how student positioning shifted in relation to the way they established resources, operations, and outcomes in algebra II material. The findings provided an overview of the exchange of knowledge and actions within each group of students, as well as a summary of the frequency with which each student engaged in various negotiation movements. The researchers then described the interactions within each group in greater detail, highlighting the ways in which students were positioned as experts or novices through their interactions. Although positioning is inherently a dynamic process, the researchers found variations in the extent to which students repositioned themselves during problem-solving processes. Nonetheless, this case did not reveal the specific causes leading to changes in positioning during group discussions.

The research conducted by Zhang et al. (2019) investigated the interactions of four students within a collaborative mathematics problem-solving group focused on geometry, using interactive positioning theory, which includes initiation (S4), response (S2), evaluation (S1 & S3), and non-interactive (none). The findings revealed that S3 and S4 were the dominant pair in the group, where their positioning interactions demonstrated how they interacted in a coordinated initiation-response (evaluation) pattern. Additionally, S2 experienced a significant positional change from initiation to non-interactive, leading to the emergence of self-talk, while S1 played a larger role in the evaluation position during the discussion. This indicates a reciprocal relationship between students' interactive positions and group problem-solving in the context of collaborative problem-solving. However, this case did not specifically indicate the causes of the changes in positioning during group discussions.

Daher (2020) investigated the positions and emotions of student groups utilizing technology to explore geometry. The results indicated that group leaders (experts) assumed positions through knowledge, actions, initiation, perseverance, and metaprocesses, whereas followers (facilitators and novices) positioned themselves by responding to the group leader's requests for actions and by answering questions. The most distinguishing feature of collaborators was their communication with other group members, leading to reciprocal relationships among them. In this case, no changes in group discussions were observed.

Furthermore, the research by Campbell and Hodges' (2020) applied positioning theory to examine how middle and university students participated in groups to solve algebra tasks, referring to collaborative structures such as participation patterns. Thirty students at two locations (secondary school and university) were recorded using video/audio while working in groups of three on challenging math tasks. The analysis focused on how students were positioned and the narratives that developed while working in groups. The findings revealed five narratives that expressed five participation patterns exhibited by the groups: one member's confirmation, competitive strategies, free-for-all, joint construction, and collaboration between two members. In this case, the study did not reveal the causes of changes occurring in group discussions. Based on the descriptions of several research results related to positioning above, the researchers have not deeply investigated the changes in positioning of each group member and how these changes occur. Therefore, the researcher aims to explore the causes of changes in positioning during algebra problem-solving discussions.

Positioning in group discussions is a crucial activity to support mathematical problem-solving. Several theories related to problem-solving, including Gagné and Smith (1962), assert that problem-solving is a type of higher-order thinking learning that is more complex than other types of thinking. Pimta et al. (2009) highlight that problem-solving can be used as a means to develop logical thinking, build mathematical ideas, and emphasize the development of mathematical thinking skills. Anderson (2009) also states that problem-solving is a life skill that involves processes of analyzing, interpreting, reasoning, predicting, evaluating, and reflecting, which are the primary goals of various mathematics curricula across different countries.

Since problem-solving is a type of higher-order thinking learning, mathematics often requires these problem-solving skills, and it is not an easy task for educators to foster creative thinking while continually designing lessons that support the development of problem-solving skills (Chong & Shahrill, 2016; Simpol et al., 2018). One framework for problem-solving was proposed by Polya (1973) whose strategies have been recognized by many researchers as stages used in solving algebraic problems. Polya (1973) proposed four stages for problem-solving:

(1) understanding the problem,

(2) devising a plan,

(3) carrying out the plan, and

(4) looking back (Lederman, 2009; Lee, 2017; Okafor, 2019; Simpol et al., 2018; Tohir et al., 2020).

These four steps then become the problem-solving process or mental process that individuals verbalize, as defined in commognitive terms.

In this context, commognitive is used as a lens to observe the cognitive processes and communication of individuals in cognitive activities. Cognitive activities, such as commognitive, are analyzed by several researchers concerned with the study of mental processes (cognitive) and the conveyance of information to oneself or others, conducted either verbally or non-verbally. They consist of four main components: word

use, visual mediators, endorsed narratives, and routines (Caspi & Sfard, 2012; Kim et al., 2017; Sfard, 2001, 2006, 2008, 2015; Sfard et al., 1998; Sriraman, 2009; Viirman, 2015). These four commognitive components have also been studied with a primary focus on prospective teachers (Nardi et al., 2014; Tuset, 2018; Viirman, 2015; Zayyadi et al., 2019, 2020).

Nardi et al. (2014) investigated effective communication through the analysis of word use and visual mediators in the context of problem-solving in small groups, analyzing variations in definitions and commognitive conflicts during the transition from secondary school to university. Viirman (2015) studied three categories–providing explanations, motivation, and posing questions–in teaching from a commognitive perspective. Tuset (2018) conducted research aimed at examining how a commognitive framework can provide insights into pre-service teachers' instruction to achieve mathematics learning objectives. The research by Zayyadi et al. (2019) demonstrated that, within the commognitive framework, subjects tended to use mathematical terminology and visual mediators during the problem-understanding phase, while narratives and routines were more prevalent during the exploration and application of strategies. Furthermore, Zayyadi et al. (2020) aimed to describe the content and pedagogical content knowledge skills of prospective teachers in mathematics education from a commognitive perspective.

Based on the results of several studies related to positioning mentioned above, there has yet to be an in-depth examination of the role changes of each group participant using Polya's problem-solving strategy and commognitive in their research. Therefore, it is important for researchers to observe the causes of positioning changes, analyzed through Polya's strategy, commognitive components, and participant interactions in discussions, leading to the research question: what are the causes of changes in student positioning during group discussions utilizing Polya's strategy and commognitive? The findings of this study contribute by helping students solve algebra problems using Polya's strategy and the commognitive lens to observe the causes of changes in positioning during group discussions, based on the guidelines for participant movements in group discussions (DeJarnette & González, 2015).

METHOD

Approach and Participants

This qualitative research employs a descriptive approach, aiming to provide a detailed explanation of the causes of changes in student positioning during group discussions utilizing Polya's problem-solving strategy and commognitive components. The study involved 30 students, from which 6 were selected to form 2 groups as participants in the discussions, representing roles as experts, facilitators, and novices. The participant selection technique included:

(a) potential participants solving problems individually,

(b) the researcher analyzing each participant's answers,

(c) the researcher interviewing each prospective participant, and

(d) the researcher categorizing potential participants based on their mathematical ability and interview results (high, medium, and low).

The participants were mathematics education students from STKIP Taman Siswa Bima, West Nusa Tenggara, Indonesia, who participated voluntarily.

Data Collection

Data were obtained from participants through tests, interviews, and observations of interactions during group discussions. Participants were asked to solve provided algebra problems within their groups. The test data obtained were analyzed according to the stages of Polya's problem-solving process and the commognitive components that emerged, as well as observing interactions in group discussions based on positioning and the roles played.

The problem-solving questions provided to the discussion participants are shown in **Table 1**.

Table 1. Algebra problem-solving questions

Airplane	Leave	Arrive	Time	Price (IDR)
Monday, 17 October 2022				
Lion Air	Jakarta	Surabaya	05:30-07:00	830,600.00
Super Jet Air			04:45-06:15	875,100.00
Citilink			06:00-07:30	999,000.00
Citilink	Surabaya	Makassar	06:00-08:25	1,234,325.00
Lion Air			14:30-16:00	1,114,700.00
Batik Air			13:00-14:30	1,226,200.00
Airport hotels	Makassar		1 night	800,000.00
Tuesday, 18 October 2022				
Lion Air			01:30-06:05	2,410,500.00

Table 1 (Continued). Algebra problem-solving questions

Airplane	Leave	Arrive	Time	Price (IDR)
Citilink	Makassar	Jayapura	05:00-09:25	2,410,393.00
Batik Air			04:20-09:00	2,506,000.00
Tuesday, 18 October 2022				
Batik Air	Jakarta	Surabaya	05:30-07:00	784,700.00
Lion Air			05:30-07:00	775,100.00
Super Jet Air			04:45-06:15	765,200.00
Citilink	Surabaya	Makassar	06:00-08:25	1,172,165.00
Super Jet Air			13:00-15:35	1,049,000.00
Lion Air			15:00-17:30	1,053,300.00
Airport hotels	Makassar		1 night	800,000.00
Wednesday, 19 October 2022				
Batik Air	Makkasar	Timika	04:40-08:45	2,238,100.00
Sriwijaya Air			03:05-07:00	2,292,300.00
Garuda Ina			02:35-06:40	2,382,500.00
Lion Air	Timika	Jayapura	07:00-08:05	550,600.00
Garuda Ina			07:25-08:50	707,600.00
Batik Air			09:25-10:40	643,400.00

Note. Two tourists from Jakarta are heading to Jayapura using different types of aircraft. Tourist I departs from Jakarta on Monday and stays overnight in Makassar, while Tourist II departs on Tuesday and also stays in Makassar. Based on the information provided, determine which of the two tourists is more efficient. Please explain your answer!

Table 2. Student movement guidelines during group discussions (adapted from DeJarnette & González, 2015)

Symbol	Information	Movement	Guidelines
K1	*Prior knowledge*	Provision of information	Students make statements to provide information
dK1	*Delayed primary knowledge*	Giving stimulus	Students delay providing information, and students ask for confirmation of the suggestions submitted
rK1	*Repeat K1*	Repeating K1	Students restate information from K1
K2	*Secondary knowledge*	Request information	Students ask questions to request information
rK2	*Response K1*	Response to K1	Students respond to K1 information as a follow-up to K2
A1	*Prior actor*	Taking action	Students take action without a request
dA1	*Delayed prior actor*	Apply	Students recommend themselves to take action
A2	*Secondary actor*	Action requests	Students ask others to take action, students ask questions related to having done or not doing assignments
rA2	*Response A2*	Response to A2	Students give follow-up responses to A2
Q1	*Confused response*	Confused response	Student revealed that he did not understand the information
P1	*Controlling discussion*	Provision of control of discussion activities	Students remind that time is almost over, and students ask for discussion until which part of the task
X1	*Inappropriate response*	Inappropriate response	Students respond inappropriately to requests for confirmation, statements, or questions

Table 3. Student positioning guidelines in group discussions (adapted from DeJarnette & González, 2015)

Position	Guidelines
Expert	Students often do K1 movements, students occasionally do K2 movements, students often do rK2 movements, students often do dK1 movements, students occasionally do A1 movements, students often do dA1, rA2 movements
Facilitator	Students often do K2, A1, A2, rK1 and P1 movements
Novice	Students often do K2 movements, students often do X1 and Q1 movements

Table 2 shows student movement guidelines during group discussions.

Table 3 shows student positioning guidelines in group discussions.

Data Analysis

The data analysis in this study focuses on algebra problems, which require students to solve non-routine problems related to open statements (ratios) using the four steps of Polya's problem-solving strategy. Each step of this strategy contains several commognitive components, including word use, routines, visual mediators, and endorsed narratives. The activities for solving algebra problems were conducted collaboratively, involving cognitive interactions resulting from the placement of group members, which consisted of expert participants, facilitators, and novices,

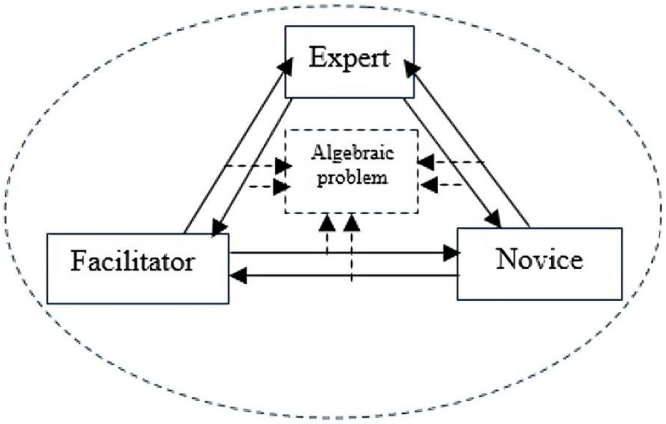


Figure 1. Patterns of cognitive interaction that contain positioning changes in solving algebra problems (adapted from DeJarnette & González, 2015)

as illustrated in **Figure 1**. Cognitive interactions during collaborative problem-solving, based on participant placements, were observed through video recordings and were further confirmed through interviews to validate the collaborative problem-solving outcomes. The results of the collaborative interviews were transcribed to depict changes in student positioning during collaborative problem-solving and their underlying causes.

The data analysis focused on the algebra problems presented to the discussion groups, with responses analyzed based on the steps of Polya's problem-solving strategy and commognitive components. Additionally, it involved analyzing the activities and positioning interactions conducted during group discussions based on observation results/videos, as well as transcribing the interview data. The findings obtained were then used to describe the causes of changes in student positioning during discussions of algebra problem-solving.

RESULTS

The results of problem-solving from the two student discussion groups, namely group 1 and group 2, are outlined as follows. This research not only focuses on the outcomes of the given problem-solving tasks but also on the activities and interactions of participants within the groups. Each group consists of three members, with participants taking on the roles of expert, facilitator, and novice. During the group discussions, both Polya's problem-solving steps and the commognitive aspects of the participants were observed. Participants were tasked with forming groups and working on the provided open-ended problem-solving questions. Group 1 participants were coded as EP1 (expert), FP1 (facilitator), and NP1 (novice), while group 2 participants were coded as EP2 (expert), FP2 (facilitator), and NP2 (novice). Each group was given two identical questions, with each question lasting 25 minutes. **Table 4** displays the frequency distribution of K1, rK1, dK1, K2, rK2, A1, dA1, A2, rA2, Q1, P1, and X1 during participant interactions.

Based on **Table 4**, it can be seen that in group 1, NP1 presented rK1 (reiterating information from K1) with a percentage of 50%, K2 (asking to obtain information) with a percentage of 25%, Q1 (expressing that he does not understand the information) with a percentage of 100%, and X1 (providing an inappropriate response) with a percentage of 100%. Meanwhile, FP1 frequently engaged in K1 (asking questions to provide information) with a percentage of 73%, dK1 (seeking confirmation of the proposed suggestion) with a percentage of 86%, K2 (asking to obtain information) with a percentage of 25%, rK2 (responding to information from K1 as a follow-up to K2) with a percentage of 67%, A1 (taking action without a request) with a percentage of 100%, dA1 (recommending to take action himself) with a percentage of 100%, A2 (asking others to take action) with a percentage of 50%, and rA2 (providing a subsequent response for A2) with a percentage of 67%. At the same time, EP1 performed K1 (asking questions to provide information) with a percentage of 27%, rK1 (reiterating information from K1) with a percentage of 50%, dK1 (seeking confirmation of the proposed suggestion) with a percentage of 14%, K2 (asking to obtain information) with a percentage of 50%, rK2 (responding to information from K1 as a follow-up to K2) with a percentage of 33%, A2 (asking others to take action) with a percentage of 50%, rA2 (providing a subsequent

Table 4. Frequency distribution of participants

Code	K1	rK1	dK1	K2	rK2	A1	dA1	A2	rA2	Q1	P1	X1
NP1		4		1						2		1
FP1	8		6	1	4	1	1	1	2			
EP1	3	4	1	2	2			1	1		1	
NP1		50%		25%						100%		100%
FP1	73%		86%	25%	67%	100%	100%	50%	67%			
EP1	27%	50%	14%	50%	33%			50%	33%		100%	
NP2				2						6		1
FP2	1	2		7	2			4				
EP2	9		3	2	3				1		1	2
NP2				18%						100%		33%
FP2	10%	100%		64%	40%			100%				
EP2	90%		100%	18%	60%				100%		100%	67%

Table 5. Frequency distribution of the number of exchanges and actions of participants

Participants	Knowledge exchange	Action exchange
NP1	6	2
FP1	19	5
EP1	12	3
NP2	7	2
FP2	12	4
EP2	15	5

response for A2) with a percentage of 33%, and P1 (reminding that time is nearly up) with a percentage of 100%.

For group 2, NP2 engaged in K2 (asking to obtain information) with a percentage of 18%, Q1 (expressing that he does not understand the information) with a percentage of 100%, and X1 (providing an inappropriate response) with a percentage of 33%. FP2 performed K1 (asking questions to provide information) with a percentage of 10%, rK1 (reiterating information from K1) with a percentage of 100%, K2 (asking to obtain information) with a percentage of 64%, rK2 (responding to information from K1 as a follow-up to K2) with a percentage of 40%, and A2 (asking others to act) with a percentage of 100%. Meanwhile, EP2 engaged in K1 (asking questions to provide information) with a percentage of 90%, dK1 (seeking confirmation of the proposed suggestion) with a percentage of 100%, K2 (asking to obtain information) with a percentage of 18%, rK2 (responding to information from K1 as a follow-up to K2) with a percentage of 60%, rA2 (providing a subsequent response for A2) with a percentage of 100%, P1 (reminding that time is nearly up) with a percentage of 100%, and X1 (providing an inappropriate response) with a percentage of 67%.

Additionally, in the important positioning, the exchange of roles and knowledge should also be noted. The frequency distribution of role exchanges and actions among discussion participants is presented in **Table 5**.

The following presents the problem-solving and discussions from both groups.

Group 1

Group 1 began their discussion to understand the given problem. At 01:10, FP1 took the lead in opening the discussion by reading the problem aloud, while the other group members listened. Then, at 03:00, FP1 invited the other group members to discuss the given problem. At 03:05, EP1 started explaining to the others in the group what steps should be taken to solve the problem. EP1 suggested illustrating two tourists with the symbols W1 and W2, while FP1 illustrated two different airplanes with the symbols P1 and P2. The goal of EP1 and FP1 was to simplify the next steps in the solution. After that, EP1 requested further suggestions from other members; there was a moment of silence among the group

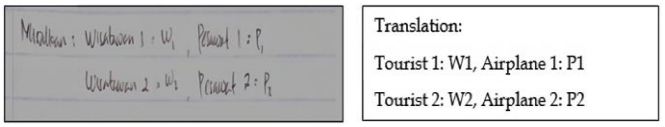


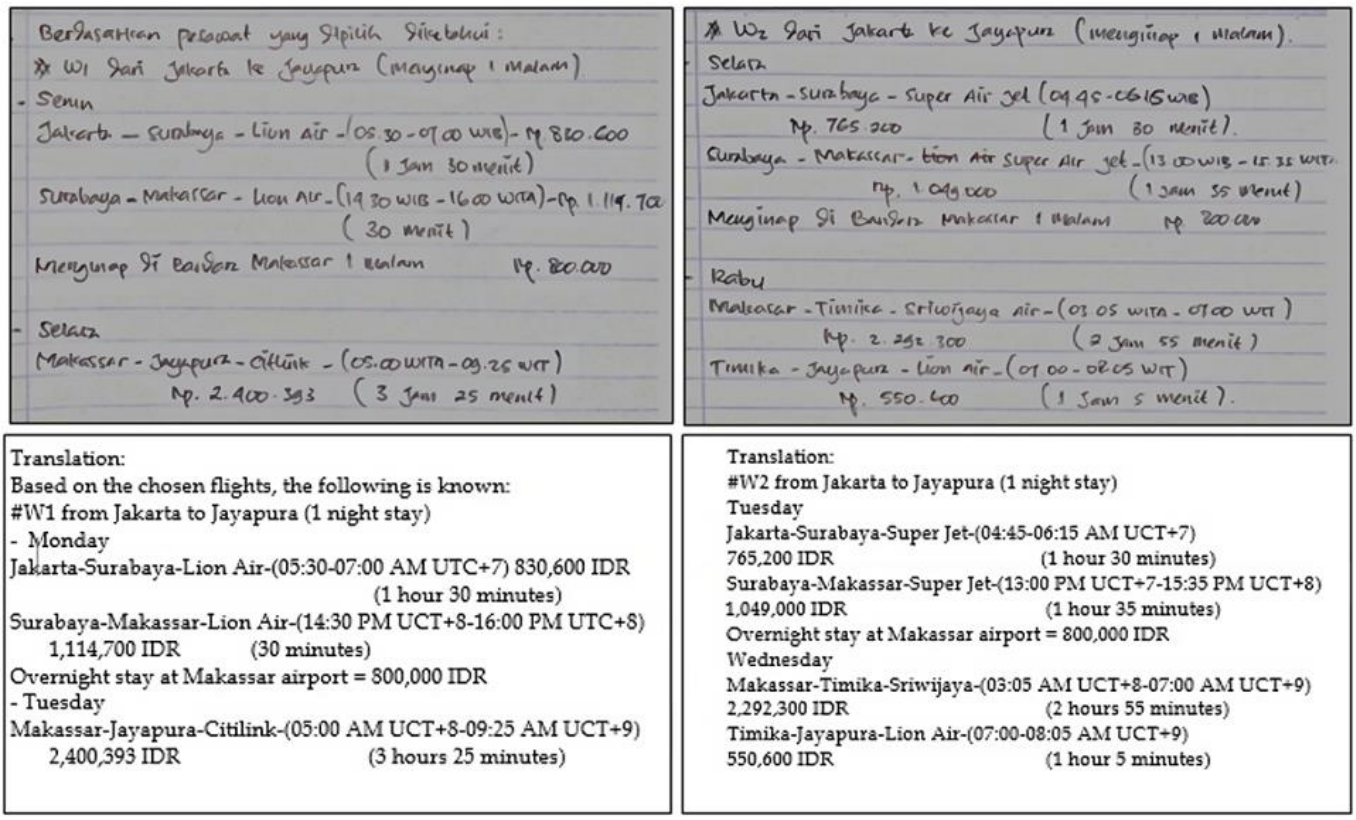
Figure 2. Stages of understanding the problem (Source: Authors' own elaboration)

members until NP1 asked at 03:10 whether the symbols W represented tourists, and the symbol P represented airplanes. At 03:11, EP1 and FP1 confirmed this, and they all agreed on it. EP1 wrote down this representation from 03:15 to 04:30, as shown in **Figure 2**.

During the participants' interactions in the group discussion, group 1 was able to express ideas for problem-solving, creating or generating new ideas or perspectives regarding mathematics. Based on the discussions and interviews conducted by the researcher, EP1, FP1, and NP1 were engaged in the stages of understanding the problem. EP1 began by establishing variables for the tourists, namely W1 and W2, and requested approval from FP1 and NP1 for these symbols. FP1 agreed and added that the airplanes also needed symbols, namely P1 and P2, and sought approval again. NP1, who initially needed clarification on the meanings of these symbols, eventually agreed after receiving a clear explanation. At this stage, group 1 introduced word use (WU) and iconic visual mediators (VMI). This process illustrates how they systematically understood and agreed upon the representation of the problem before moving on to the next stage, in line with Polya's fundamental principles of problem-solving.

Then, at 04:32, FP1 redirected the group to continue working on the answers. At 04:35, EP1 began and invited the group members to complete the task together. EP1 explained that they would differentiate between W1 and W2, starting by determining the time for W1's journey from Jakarta to Jayapura from Monday to Tuesday with stops at two locations, namely Surabaya and Makassar, and an overnight stay in Makassar. Only then would they determine W2's schedule from Jakarta to Jayapura on Tuesday and Wednesday with stops at three locations: Surabaya, Makassar, and Timika. FP1 and NP1 listened attentively to EP1's explanation while writing down what was being discussed, which took them up to 20:00. The results of their group's work are presented in **Figure 3**.

In the planning stage of problem-solving, group 1 smoothly developed their plan, where EP1 guided their group members to discuss while writing on the answer sheet. During this stage, from 05:00 to 20:00, while discussing, EP1 began to write about tourist 1 traveling from Jakarta to Jayapura. Since tourist 1 departs on Monday, they focused on the journey for that day. The three of them examined **Table 5** in the question to determine which flight from Jakarta to Surabaya was more time and cost-efficient. FP1 chose Lion Air at 05:30 - 07:00 AM UCT+7, which takes 1 hour and 30 minutes,



Translation:
Based on the chosen flights, the following is known:
#W1 from Jakarta to Jayapura (1 night stay)
- Monday
Jakarta-Surabaya-Lion Air-(05:30-07:00 AM UTC+7) 830,600 IDR
(1 hour 30 minutes)
Surabaya-Makassar-Lion Air-(14:30 PM UCT+8-16:00 PM UTC+8)
1,114,700 IDR (30 minutes)
Overnight stay at Makassar airport = 800,000 IDR
- Tuesday
Makassar-Jayapura-Citilink-(05:00 AM UCT+8-09:25 AM UCT+9)
2,400,393 IDR (3 hours 25 minutes)

Translation:
#W2 from Jakarta to Jayapura (1 night stay)
Tuesday
Jakarta-Surabaya-Super Jet-(04:45-06:15 AM UCT+7)
765,200 IDR (1 hour 30 minutes)
Surabaya-Makassar-Super Jet-(13:00 PM UCT+7-15:35 PM UCT+8)
1,049,000 IDR (1 hour 35 minutes)
Overnight stay at Makassar airport = 800,000 IDR
Wednesday
Makassar-Timika-Sriwijaya-(03:05 AM UCT+8-07:00 AM UCT+9)
2,292,300 IDR (2 hours 55 minutes)
Timika-Jayapura-Lion Air-(07:00-08:05 AM UCT+9)
550,600 IDR (1 hour 5 minutes)

Figure 3. Stages of planning problem-solving (Source: Authors' own elaboration)

while EP1 selected Super Jet Air at 04:45 - 06:15 AM UCT+7, which also takes 1 hour and 30 minutes. However, FP1 suggested they review the costs of both airlines, as Lion Air costs 830,600 IDR and Super Jet costs 875,100 IDR; they decided to choose the cheaper option, which was Lion Air. They continued in this manner, as they had identified a pattern, agreeing to complete this stage until they reached Jayapura Airport. At this stage, negotiations took place during the group discussion. Below is the transcript of the conversation that represents the negotiation events:

EP1: (Suggesting to other participants to consider the travel of tourist 1 from Jakarta to Jayapura). Let's observe which travel option from Jakarta to Jayapura is more economical in terms of time and cost? [K2].

FP1: For me, it's Lion Air at 05:30-07:00 [K1], which takes 1 hour and 30 minutes [K1].

EP1: For me, it's Super Jet at 04:45-06:15 [K1], which also takes 1 hour and 30 minutes [K1].

NP1: Why is that? [X1], why is it different? [Q1].

FP1: Please pay attention to the costs [A1]; Lion Air is 830,000 IDR [K1], while Super Jet is 875,100 IDR [K1].

EP1: Oh, you're right, hehehe [rK1].

FP1: So for tourist 2, the process is the same as tourist 1 [K1], right? [dK1].

EP1: Yes, that's correct [rK1].

FP1: Ok, let's examine the time differences between UCT+7, UCT+8, and UCT+9 [dA1], each being 1 hour apart, right? [K2].

EP1: Yes, UCT+7 is 1 hour faster than UCT+8 [rK2], and similarly, UCT+8 is 1 hour faster than UCT+9 [rK2].

FP1: So, UCT+7 is 2 hours faster than UCT+9 [rK2]. What about you, NP1? [dK1].

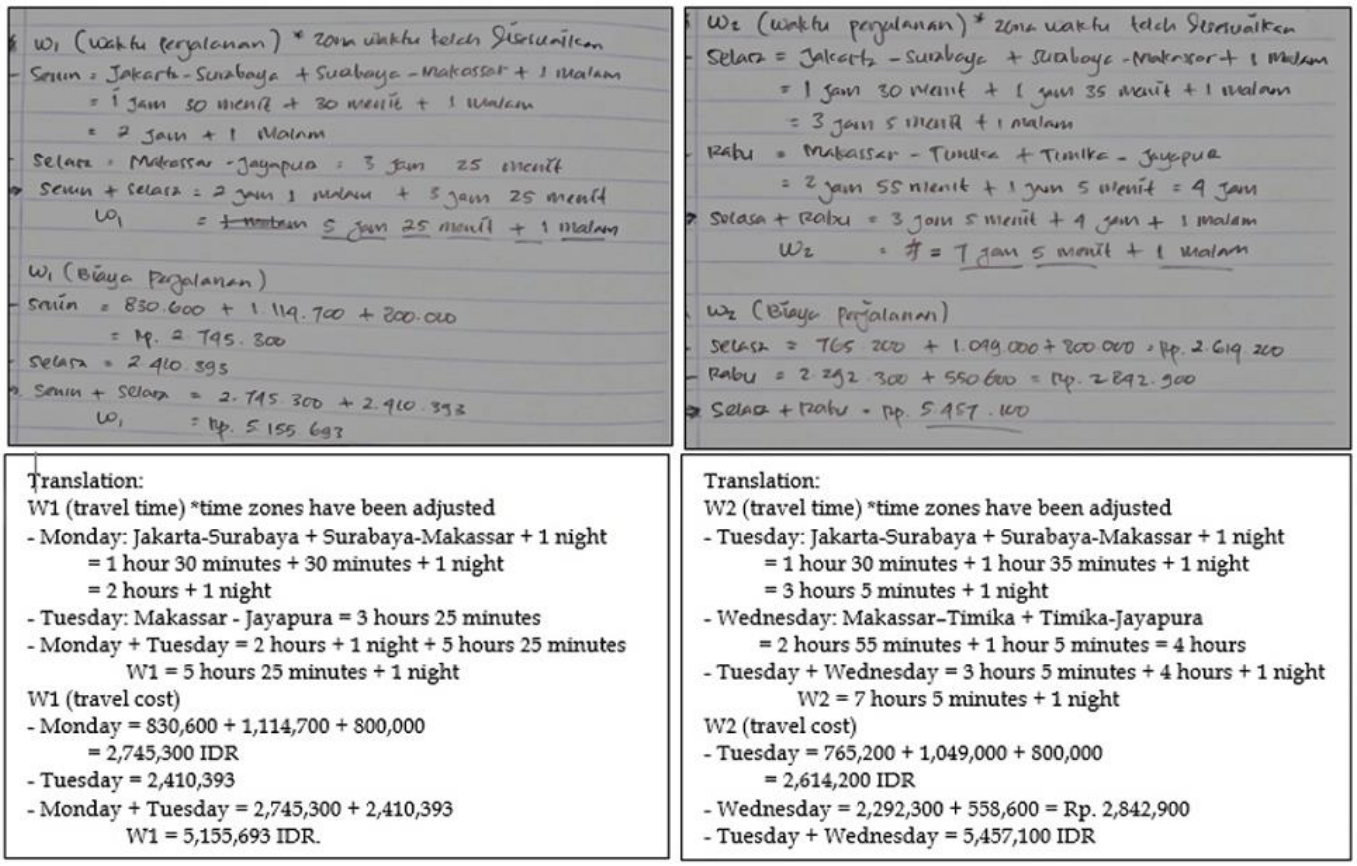
NP1: I'm going along with you guys, hehehe [rK1].

The researcher interviewed group 1 in accordance with the results of their work presented in **Figure 2** and the discussions that took place:

R: Why did you list it that way? (pointing to **Figure 3**).

EP1, FP1: So that we can distinguish between the journeys of tourist 1 and tourist 2, sir, and it helps us determine the next steps.

R: Okay. Do you know the difference between UCT+7, UCT+8, and UCT+9?



Translation:
W1 (travel time) *time zones have been adjusted
- Monday: Jakarta-Surabaya + Surabaya-Makassar + 1 night
= 1 hour 30 minutes + 30 minutes + 1 night
= 2 hours + 1 night
- Tuesday: Makassar - Jayapura = 3 hours 25 minutes
- Monday + Tuesday = 2 hours + 1 night + 5 hours 25 minutes
W1 = 5 hours 25 minutes + 1 night
W1 (travel cost)
- Monday = 830,600 + 1,114,700 + 800,000
= 2,745,300 IDR
- Tuesday = 2,410,393
- Monday + Tuesday = 2,745,300 + 2,410,393
W1 = 5,155,693 IDR.

Translation:
W2 (travel time) *time zones have been adjusted
- Tuesday: Jakarta-Surabaya + Surabaya-Makassar + 1 night
= 1 hour 30 minutes + 1 hour 35 minutes + 1 night
= 3 hours 5 minutes + 1 night
- Wednesday: Makassar–Timika + Timika-Jayapura
= 2 hours 55 minutes + 1 hour 5 minutes = 4 hours
- Tuesday + Wednesday = 3 hours 5 minutes + 4 hours + 1 night
W2 = 7 hours 5 minutes + 1 night
W2 (travel cost)
- Tuesday = 765,200 + 1,049,000 + 800,000
= 2,614,200 IDR
- Wednesday = 2,292,300 + 558,600 = Rp. 2,842,900
- Tuesday + Wednesday = 5,457,100 IDR

Figure 4. Stages of problem-solving (Source: Authors' own elaboration)

> EP1, FP1: Yes, sir, we do. The difference between UCT+7 and UCT+8, and UCT+8 and UCT+9 is each 1 hour, while from UCT+7 to UCT+9 it is 2 hours, sir.

Based on the transcript of the discussion and interviews, EP1, FP1, and NP1 underwent the stage of planning strategy according to Polya's approach. EP1 began by suggesting that they check the travel options from Jakarta to Jayapura to determine which one was more time and cost-efficient. FP1 and EP1 compared two flight options, Lion Air and Super Jet, focusing on the timing and costs. When NP1 inquired about the differences, FP1 explained the cost discrepancies between the two flights, which helped NP1 better understand the situation. Furthermore, FP1 directed their attention to the time zone differences between UCT+7, UCT+8, and UCT+9, ensuring that all members understood that the time difference from UCT+7 to UCT+8 and UCT+8 to UCT+9 is one hour, and two hours from UCT+7 to UCT+9. After this clarification, EP1 and FP1 decided that the process for tourist 2 would be the same as for tourist 1.

Group 1 introduced word use (WU) and iconic visual mediators (VMI) by listing all the travel routes they deemed efficient in terms of both cost and time, creating a format that resembled a table. This plan would facilitate finding solutions to the existing problems. Additionally, the members of group 1 effectively utilized endorsed narrative (EN) during their group discussions, allowing them to identify patterns in determining the route and costs of travel, which aided in deciding the next steps. In this section, a negotiation process occurred between EP1 and FP1, where FP1 occasionally took over the discussion by acting without being prompted (prior actor [A1]), self-recommending actions (delayed prior actor [dA1]), and strongly defending opinions, thus assuming the role of an initiator that fostered self-efficacy.

Meanwhile, EP1 took the opposite approach, mainly requesting FP1 to act (secondary actor [A2]), and his opinions could not be upheld (were rejected by other participants), which resulted in EP1, initially playing the role of an initiator, transforming into a respondent, leading to self-talk.

With this understanding, they agreed to proceed with the following steps in accordance with their strategic planning results. This process reflects the stages of strategizing in problem-solving, where they identified the relevant variables, understood the differences and relationships among those variables, and planned the next steps based on that understanding.

Group 1, from minute 21:01 to minute 40:00, continued their discussion and completed their work in the stage of implementing problem-solving. The results of their group discussion at this stage can be seen in **Figure 4**.

In this stage of implementing problem-solving, FP1 took the lead in the discussion by explaining to the other participants the steps for resolution, starting with the time taken followed by the costs incurred. Tourist 1 departed on Monday from Jakarta to Surabaya and then from Surabaya to Makassar, staying overnight in Makassar. The time spent was 2 hours plus the overnight stay of 1 night. At minute 21:05, NP1 raised a question about why it was only 2 hours. FP1 responded that this was because they had adjusted the time at the beginning. Then, EP1 continued to write at minute 21:08 the time used by tourist 1 on Tuesday from Makassar to Jayapura, which was 3 hours 25 minutes. Subsequently, EP1 asked FP1 and NP1 about the next steps, and FP1 explained at minute 21:12 that they would accumulate the time used by tourist 1 on Monday and Tuesday. EP1 acknowledged this, and by minute 21:17 they calculated, resulting in a total time used by tourist 1 of 5 hours 25 minutes plus 1 night of accommodation.

After that, at minute 21:20, they began to determine the costs incurred by tourist 1 on Monday and Tuesday. FP1 directed them to revisit their answer sheet, and EP1 started writing down the costs for Monday, which amounted to 2,745,300 IDR, and for Tuesday, the expenses were 2,410,393 IDR, resulting in a total of 5,155,693 IDR. Then, at minute 21:25, they applied the same method for tourist 2, completing the resolution steps at this stage, where the time spent by tourist 2 on Tuesday and Wednesday was 7 hours 5 minutes plus an overnight stay in Makassar. The expenses incurred by tourist 2 totaled 5,457,100 IDR. Group 1 completed this step at minute 40:00. During tourist 1's overnight stay in Makassar, a serious discussion emerged, as EP1 wanted to convert the time into hours or minutes, but FP1 insisted on using just 1 night and defended this viewpoint, arguing that it would simplify the calculations, and they eventually reached an agreement.

At this stage, negotiations were still occurring within group 1, as reflected in the following transcript of their discussion:

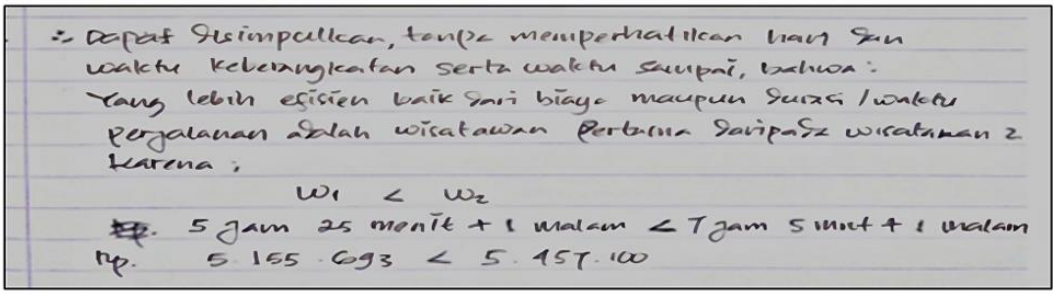
> FP1: Please, EP1, start the discussion [A2].
>
> EP1: No, let FP1 start [rA2].
>
> FP1: Alright, let's pay attention to the overnight stay of the tourists in Makassar [K1]. How about we just use 1 night without converting it to hours or minutes? [dK1].
>
> EP1: Why not convert it to hours or minutes to make it easier? [K2].
>
> FP1: Actually, I think it's easier without converting it [rK2]. What do you think? [dK1].
>
> NP1: How can it be easier? [Q1].
>
> FP1: Yes, because in the end, we're just adding 1 night [K1]. If we convert it, I'm worried there might be mistakes later [rK2].
>
> EP1: Okay, agreed [rK1].
>
> NP1: I'll go along with you guys [rK1].

Based on the work presented in **Figure 4** and their conversation, the researcher conducted interviews, resulting in the following transcript:

> R: How did you come up with the idea to solve the problem as shown in **Figure 4**?
>
> FP1: We started by separating tourist 1 and tourist 2. For tourist 1, we determined the time for both Monday and Tuesday before calculating the costs. The same approach was taken for tourist 2 on Tuesday and Wednesday.
>
> EP1: However, we did have some differing opinions earlier about whether to keep it as 1 night or convert it to hours or minutes for the overnight stay in Makassar, but we eventually agreed not to change it.

Based on the transcript of the discussion and the interviews conducted during the implementation stage of the problem-solving process, the discussion within group 1 illustrates how they addressed differences of opinion and reached a consensus. FP1 initiated by suggesting that they retain the overnight stay of the tourists in Makassar as 1 night without converting it to hours or minutes. EP1 proposed changing it to make the calculations easier; however, FP1 argued that maintaining the time would reduce the risk of errors in the final calculations. NP1 questioned FP1's reasoning, and after hearing the explanation that adding 1 night is simpler than converting it, EP1 and NP1 agreed with FP1's decision. In the interview, FP1 explained that they started by separating the calculations for tourist 1 and tourist 2, as well as determining the time and costs for each day of the trip. EP1 added that despite the differing opinions on how to calculate the overnight stay duration, the group ultimately agreed not to change it from one night.

It can be said that they successfully solved the given problem, and they were even able to differentiate between the time used and the costs incurred by each of the tourists, 1 and 2. Observing this, group 1 employed routine exploratory (RE) in tackling the problem, which involved explaining how to approach and solve the issue while articulating when to utilize specific procedures. Furthermore, they also utilized routine corrigibility (RC), checking the explanations or narratives behind the reasons for using certain procedures or methods. In this stage, the endorsed narrative (EN) they constructed was very effective, as they followed the planned procedures



Translation:
In conclusion, irrespective of the day and time of departure and arrival, it can be stated that the first tourist is more efficient in terms of both cost and travel duration than the second tourist because:
W1 < W2
5 hours 25 minutes + 1 night < 7 hours 5 minutes + 1 night
5,155,693 IDR < 5,157,100 IDR

Figure 5. Stages of review (Source: Authors' own elaboration)

step by step in their work. This process reflects the implementation stage of problem-solving where the group decides on the best approach through discussion, clarification, and consensus. They then proceed with the agreed-upon steps to resolve the problem. This collaborative effort highlights the importance of communication and shared understanding among group members in effectively addressing challenges and reaching satisfactory solutions.

In the final stage, which is the review stage, group 1 examines their work from the beginning, starting from minute 41:00 to minute 50:00. After FP1 and the other group members ensure that their work is correct, they provide a conclusion as shown in **Figure 5**.

From **Figure 5**, it is evident that group 1 concludes that in terms of both time and cost efficiency, tourist 1 is more favorable, as tourist 1 requires less time and incurs lower costs compared to tourist 2.

Based on the discussion and interview during the evaluation or review stage, group 1 concluded their analysis and verified those conclusions. In the final discussion, EP1 proposed that they summarize their findings as time was running out, and FP1 agreed that tourist 1 is more efficient than tourist 2 in terms of both time and cost. EP1 and NP1 concurred with this conclusion. In the interview, EP1 explained that the group's conclusion is that tourist 1 is more efficient in terms of time and expenses. FP1 elaborated that this conclusion is based on a clear analysis evident from their work results, where the time and costs for tourist 1 are lower compared to tourist 2. This is reflected in **Figure 4**, which indicates that tourist 1 is the more efficient choice.

Group 1 effectively utilized VMS and EN, demonstrating their ability to explain reasons and connect objects, relationships with previous materials, and processes, such as definitions, theorems, and proofs (Sfard, 2007; Tuset, 2018). The changes in the endorsed narrative are evidenced by the accuracy in solving the given problems and the increase in the quantity of responses provided (productive) (Webb et al., 2019).

This evaluation process reflects the final stage in problem-solving, where the group reassesses their analysis results, ensures consistency with the data, and concludes the most efficient solution.

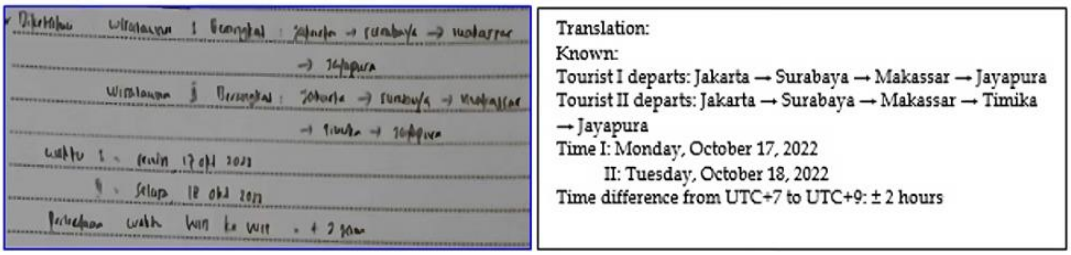
These findings are highly relevant to the research objectives that explore the changes in roles and positions of participants in the problem-solving process (**Table 6**). The shifts in positions indicate adaptation and the dynamics of group interactions, which are crucial for effective collaboration. The movement from expert to facilitator and vice versa illustrates that self-efficacy and self-talk enable group members to take on more active and productive roles. This discovery enriches our understanding of group dynamics and collaborative learning strategies in problem-solving. This group 1 is named "Transformative Trio," reflecting the dynamic changes in roles and contributions of each group member, highlighting their ability to adapt and grow throughout the collaborative problem-solving process.

Group 2

Group 2 began discussing the problem presented to them. At 01:15, EP2 took the initiative to open the discussion by reading the problem aloud, while the other group members listened attentively. Then, at 03:02, EP2 invited other members to collaboratively discuss the question at hand. At 03:04, EP2 started engaging with team members while writing down the travel routes of each tourist, noting that tourist 1 traveled from Jakarta to Surabaya, then from Surabaya to Makassar, and finally from Makassar to Jayapura. EP2 used an arrow symbol "→" to indicate the routes. Following that, at 07:10, EP2 wrote down the route taken by tourist 2, who traveled from Jakarta to Surabaya, then from Surabaya to Makassar, from Makassar to Timika, and finally from Timika to Jayapura.

Table 6. Polya and cognitive problem-solving and causes of shifts in group 1's positioning

Participant positioning			
Participant's starting position	Excellent (EP1)	Facilitator (FP1)	Novice (NP1)
Polya's strategy	Demonstrated all four steps of Polya, although NP1 only followed EP1 and FP1.		
Commognitive	Revealed all four components: word use (WU), visual mediators symbolic (VMS), visual mediators iconic (VMI), routine exploratory (RE), routine corrigibility (RC), and endorsed narrative (EN), although NP1 mostly followed other participants.		
Position shift	*Facilitator* (F'P1)	*Excelent* (E'P1)	*Novise* (NP1) (constant)
Causes of position shift	• Engaged in K1 (prior knowledge), rK1 (repeat K1), dK1 (delayed primary knowledge), K2 (secondary knowledge), rK2 (response K1), A2 (secondary actor), rA2 (response A2), P1 (controlling discussion). • Initially acted as an initiator but shifted to a response role that elicited self-talk.	• Engaged in K1 (prior knowledge), dK1 (delayed primary knowledge), K2 (secondary knowledge), rK2 (response K1), A1 (prior actor), dA1 (delayed prior actor), A2 (secondary actor), rA2 (response A2). • Initially acted as a responder and shifted to an initiator role that fostered self-efficacy.	• Engaged in rK1 (repeat K1), K2 (secondary knowledge), Q1 (confused response), X1 (inappropriate response). • Remained in the position as a novice.



Translation:
Known:
Tourist I departs: Jakarta → Surabaya → Makassar → Jayapura
Tourist II departs: Jakarta → Surabaya → Makassar → Timika → Jayapura
Time I: Monday, October 17, 2022
II: Tuesday, October 18, 2022
Time difference from UTC+7 to UTC+9: ± 2 hours

Figure 6. Stages of understanding the problem (Source: Authors' own elaboration)

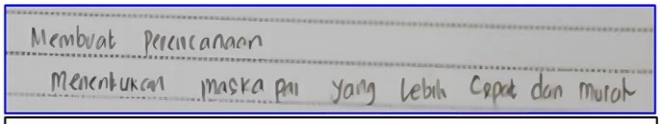
Subsequently, at 10:00, EP2 noted the departure days for tourist 1, which were Monday and Tuesday. EP2 also wrote down the time difference between UTC+7 and UTC+9, as illustrated in **Figure 6**.

During the group discussion, there was not much interaction from group 2; it was evident that EP2 dominated the discussion activity in solving the first problem while involving the other group members. From the results, group 2 appeared to have still not fully captured the information presented in the problem.

Based on the discussion and interviews during the problem-understanding phase, group 2 demonstrated how they processed and verified relevant information. EP2 initiated the discussion by inquiring about the travel routes of tourist 1 and tourist 2 from Jakarta to Jayapura and emphasized the use of the symbol "→" to illustrate these routes. FP2 confirmed this symbol, and EF2 and NP2 agreed with this decision, although NP2 felt unclear and followed the group's decision. EP2 then directed attention to the time difference from UTC+7 to UTC+9 and asked NP2 for a response, but NP2 did not provide an answer. EP2 explained that the time difference from UTC+7 to UTC+8 is 1 hour, and from UTC+8 to UTC+9 is also 1 hour, making the total time difference from UTC+7 to UTC+9 2 hours, which FP2 confirmed. In the interview, EP2 clarified that the main information in the problem pertains to the tourists' departure from Jakarta to Jayapura, while FP2 added that each tourist's journey takes 2 days. EP2 and FP2 noted that tourist 1 travels on Monday and Tuesday, while tourist 2 travels on Tuesday and Wednesday. When asked why the travel time for tourist 2 was not included in the answer, EP2 admitted that it was an oversight and recognized the importance of recording that information. This process reflects how the group understood and organized the necessary information for thorough problem analysis.

Based on the information from the discussion and interviews, it appears that during the problem-understanding phase, group 2 did not provide complete information, resulting in a lack of detailed communication of all the information presented in the problem. Nonetheless, there were instances of word use (WU) and visual mediators symbolic (VMS) introduced by group 2, represented by "→" and "±," indicating the travel routes of the tourists from one location to another. Then, at 15:00, EP2 encouraged their group members to continue solving the problem. In the planning phase of the problem-solving process, group 2 only noted information related to determining which tourist arrived at their destination faster and which incurred lower travel costs. Their group's work is presented in **Figure 7**.

At this stage, group 2 did not provide any information; they only wrote a statement to determine which option was faster and cheaper, and there was no



Translation:
Making a plan
Determining which is faster and cheaper

Figure 7. Stage of planning the problem-solving (Source: Authors' own elaboration)

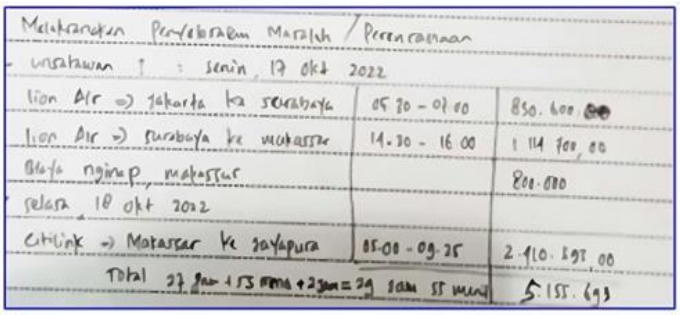
discussion activity taking place. At this point, all information should have been clearly conveyed so that the next stage would make problem-solving easier. Seeing that group 2 only recorded information as stated above, the researcher conducted interviews with them.

Based on the discussion and interviews during the strategy planning phase, group 2 discussed the steps they would take to determine which tourist arrived at their destination faster and used a smaller budget. FP2 asked whether it was sufficient to simply write the initial information as shown in **Figure 6**, and EP2 confirmed that it was enough. NP2 inquired about the reason for only recording that information, and EP2 explained that they were only providing initial information regarding the criteria for evaluating the efficiency of tourist 1 and tourist 2's travels. In the interview, EP2 reiterated that they were focusing on the preliminary information that established efficiency criteria based on time and cost. When asked why they did not create a list of times and costs for all the routes taken by the two tourists, EP2 clarified that they would directly perform the calculations in the next phase. FP2 supported this statement, indicating that they chose to simplify the initial information and would provide further details in the next step. This process illustrates how group 2 planned their problem-solving strategy by organizing non-detailed information before moving on to the subsequent stage.

At this stage, group 2 deliberately did not write down all the information related to the time and costs of tourist 1 and tourist 2's travels, resulting in the absence of a visual mediator (VM) at this point. Subsequently, from 17:00 to 38:00, group 2 engaged in group discussions again, during which EP2 encouraged the group members to focus on solving the existing problem (**Figure 8**).

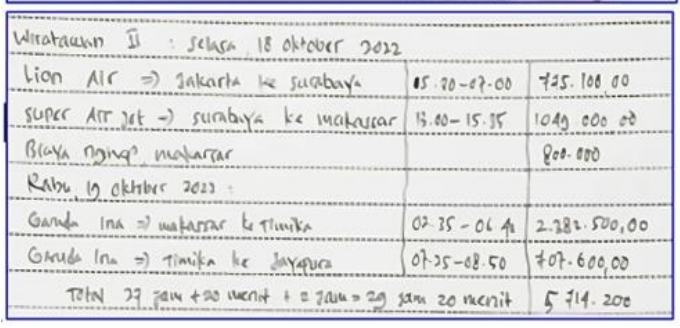
During this problem-solving stage, a shift in position occurred, where EP2 was unable to correctly answer the questions posed by other group members. The transcription of the discussion results from group 2 that took place at this stage is, as follows:

> FP2: Please, EP2 and NP2, let's discuss the next steps [A2].
>
> EP2: Alright, allow me to convey this … [rA2]. In this part, we will list tourist 1 and tourist 2, starting from Jakarta to Jayapura [K1]. Starting with tourist 1, for Jakarta to Surabaya, we chose Lion Air at 05:30-07:00, and from Surabaya to Makassar at 14:30-16:00. [K1]
>
> FP2: Why don't we take Super Jet Air, which departs earlier, at 04:45-06:15? [K2].
>
> EP2: Look at the costs, Lion Air is cheaper [K1].
>
> FP2: That's true, what about you, NP2? [K2].
>
> NP2: I'm confused [Q1].
>
> FP2: Okay, let's move on from Makassar to Jayapura [A2].



Translation:
Implementing Problem Solving
Tourist I: Monday, October 17, 2022

Lion Air ⇒ Jakarta to Surabaya	05:30-07:00	830,600
Lion Air ⇒ Surabaya to Makassar	14:30-16:00	1,114,700
Accommodation cost in Makassar		800,000
Tuesday, October 18, 2022		
Citilink ⇒ Makassar to Jayapura	05:00-09:25	2,410,393
Total 27 hours + 55 minutes		5,155,693
+ 2 hours = 29 hours 55 minutes		



Translation:
Tourist II: Tuesday, October 18, 2022

Lion Air ⇒ Jakarta to Surabaya	05:30-07:00	775,100
Super Jet ⇒ Surabaya to Makassar	13:00-15:35	1,049,000
Accommodation cost in Makassar		800,000
Wednesday, October 19, 2022		
Garuda ⇒ Makassar to Timika	05:00-09:25	2,410,393
Garuda ⇒ Timika to Jayapura	07:25-08:50	707,600
Total 27 hours + 20 minutes + 2 hours		5,714,200
= 29 hours 20 minutes		

Figure 8. Problem-solving stage (Source: Authors' own elaboration)

EP2: For the route from Makassar to Jayapura, we will take Citilink [K1].

FP2: Okay, agreed [rK1]. Let's continue to tourist 2, go ahead [A2].

EP2: For tourist 2, the journey from Jakarta to Surabaya is with Lion Air at 05:30-07:00 [K1].

FP2: Shouldn't we take Super Jet Air, which is earlier and cheaper? [K2].

EP2: Let's try this one first … [X1].

FP2: Hmmm, alright then, what do you think, NP2? [K2].

NP2: I'm following you guys [Q1].

EP2: Okay, for the journey from Surabaya to Makassar, we will take Super Jet Air since it's cheaper [K1].

NP2: Why not Citilink which has an earlier schedule? [K2].

FP2: But the flight from Jakarta hasn't even arrived in Surabaya yet [rK2].

NP2: Hmmm … Oh, I see [X1].

FP2: Now, let's discuss the route from Makassar to Jayapura. Please continue with the discussion [A2].

EP2: Since the route from Makassar to Jayapura requires a transit to Timika first, how about we take Garuda Indonesia for both trips? [K2].

FP2: Why take that? Isn't the cost from Makassar to Timika cheaper? After that, we can take Batik Air from Timika to Jayapura. What do you think, NP2? [K2].

NP2: I agree with FP2 that from Makassar to Timika we should take Batik Air because it's cheaper, but for Timika to Jayapura, I'd prefer Lion Air as it's cheaper too, right? [K2].

FP2: That's not possible, because Lion Air departs at 07:00 while Batik Air from Makassar arrives at 08:45 [rK2].

NP2: Oh, is that so ...? [Q1].

FP2: Yes, what is our decision? [K2].

EP2: I still prefer both with Garuda Indonesia, because from Makassar it departs earlier, and we would use the same plane from Timika [X1].

FP2: Alright, we'll agree on that, but I'm not so sure …

Seeing the work and discussion of group 2 at this stage, the researcher wanted to gain a deeper understanding of the answers provided by group 1 by interviewing them. Below is the transcript of the interview results between the researcher and the group:

R: Are you all confident with your answers above?

EP2: Yes, sir.

R: Let's take another look. For tourist 1 from Jakarta to Surabaya, why did you choose the time of 05:30-07:00 AM UTC+7 with the cost of 830,600 IDR when there is an airline with an earlier schedule?

EP2: (Looking over the question again) We considered the cost, which is cheaper, sir …

R: Or is there another consideration?

FP2: No, sir.

R: Let's examine the next point. Why did you choose Garuda Indonesia for the journey from Makassar to Timika at 02:30 AM UTC+8-06:40 AM UTC+9 with a cost of 2,382,500 IDR, while there is a cheaper option at 2,292,300 IDR using Sriwijaya Air?

EP2: For that, we considered the earlier time, sir. Garuda Indonesia's flight departs at 02:35 AM UTC+8-06:40 AM UTC+9.

R: Please calculate again using the time and costs I suggested above, and you will find which one is more efficient.

EP2, FP2, NP2: Hehe, yes, sir.

R: How many hours is the time difference from AM UTC+7 to AM UTC+9? Which one is faster?

EP2: It's 2 hours, sir. UTC + 7 is faster

R: Since it starts from Jakarta, should it be added or subtracted?

EP2: Subtracted, sir.

R: Why did you say it was added by 2 hours in your response?

EP2: Oh yes, sir, we made an error.

In group 2's discussion regarding the selection of flight schedules for tourist 1 and tourist 2 from Jakarta to Jayapura, there was a debate concerning the choice of

airlines and flight schedules. EP2 proposed Lion Air from Jakarta to Surabaya and Citilink from Makassar to Jayapura, but FP2 questioned why they did not choose Super Jet Air, which has an earlier flight. After considering the lower costs, the group ultimately agreed with EP2's choices. For tourist 2, they initially selected Lion Air, but again, questions arose regarding options that were both cheaper and earlier. In the end, they decided to go with the airlines suggested by EP2, citing efficiency in both time and cost as their reasons for the choice.

The results of the researcher's interview with group 2 indicated that the group had confidence in their answers, despite some errors in the calculation of time and costs. The researcher highlighted mistakes in choosing flight times and the time difference between UTC + 7 and UTC + 9. EP2 acknowledged that they made an error in their time calculations, stating that UTC + 7 time should be reduced by 2 hours when transitioning to UTC + 9. By correcting this mistake, the researcher emphasized the importance of recalculating time and costs to achieve maximum efficiency in this problem-solving process.

Group 2 carried out problem-solving using the routine corrigibility (RC) approach; however, for tourist 2 from Jakarta to Surabaya, they incorrectly determined their choice. The group appeared to utilize a routine in addressing the problem, but still made errors. Additionally, they employed a narrative that was not precise, leading to an incorrect final result in their work. They mistakenly believed that the time difference from UTC + 9 to UTC + 7 should be added by 2 hours, when it should have actually been subtracted by 2 hours. Consequently, this led to a conflict in their cognitive understanding. At this stage, EP2 provided incorrect and less convincing answers, shifting from an initial response position to that of an evaluator, which resulted in self-talk as EP2 processed the situation.

In the final stage, which is the reflection stage, group 2 reviewed their work from the beginning, specifically from minute 40:00 to minute 50:00. EP2 and the other group members provided conclusions based on the results of their work. Therefore, the conclusions are as shown in **Figure 9**.

Based on the description in **Figure 9** and the results of the discussion, group 2 concluded that tourist 2 is 35 minutes faster than tourist 1, and tourist 1 is more cost-efficient, with a price difference of 558,507 IDR. Group 1 also concluded that the more efficient option is tourist 1. Based on the data analysis and the discussion above, there are differences between group 1 and group 2 regarding their roles in positioning during group discussions, as well as the emergence of cognitive components in each of those groups.

This finding is highly relevant to the research objective of exploring changes in the roles and positions of participants in the problem-solving process (**Table 7**).

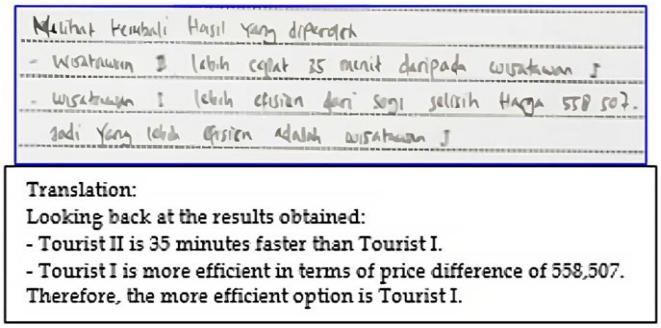


Figure 9. Stages of review (Source: Authors' own elaboration)

The shift of EP2 to an evaluator role demonstrates the dynamics and adaptability within the group when faced with errors. Despite the mistakes, the group still exhibited the use of commognitive components such as word use (WU), visual mediators (VMS), and routine corrigibility (RC). The results of this study enhance our understanding of how interactions and roles within a group can influence the effectiveness of problem-solving, offering new insights for collaborative learning strategies. Group 2 has been named "dynamic evaluators," reflecting the group's ability to adapt and critically assess during the problem-solving process, showcasing flexibility and diligence in group discussions.

DISCUSSION

Findings in Group 1 and Group 2

Group 1-Transformative trio

Findings from group 1 indicate significant dynamics in role shifts during the problem-solving process. Initially, group members were divided into excellent (EP1), facilitator (FP1), and novice (NP1). EP1 demonstrated strong understanding by applying all four steps of Polya, while NP1 tended to follow EP1 and FP1 without making significant contributions. FP1 acted as the facilitator, influencing the flow of discussion, whereas NP1 needed more encouragement to participate actively.

Over time, the positions of EP1 and FP1 changed. The initially dominant EP1 transitioned to a role more akin to that of a facilitator (F'P1), while FP1 evolved from being just a facilitator (FP1) to a position resembling excellent (E'P1). NP1 remained in the novice (NP1) position. This shift illustrates how members' roles can change in response to group dynamics and individual contributions.

The causes of this shift are linked to activities such as K1 (prior knowledge), rK1 (repetition of prior knowledge), and A2 (secondary actor). EP1, initially serving as the initiator, began to focus on other activities that led to self-talk. FP1, who started as a facilitator,

Table 7. Polya's problem solving and commognitive components, along with the causes of positioning shift in group 2

Participant positioning			
Participant's starting position	Excellent (EP1)	Facilitator (FP1)	Novice (NP1)
Polya's strategy	The three steps of Polya were identified, but there was an error in the implementation of the problem-solving process, which also affected the reflection stage.		
Commognitive	The four components emerged, namely word use (WU), visual mediators iconic (VMI), routine corrigibility (RC), but errors were still present, and endorsed narrative (EN).		
Position shift	*Facilitator* (F'P1)	Facilitator (FP1)	*Novise* (NP1) (constant)
Causes of position shift	• Engaged in activities K1 (prior knowledge), dK1 (delayed primary knowledge), K2 (secondary knowledge), rK2 (response K1), rA2 (response A2), P1 (controlling discussion), X1 (inappropriate response). • Initially acted as a response and shifted to an evaluator role, resulting in self-talk.	• Engaged in activities K1 (prior knowledge), activity rK1 (repeat K1), K2 (secondary knowledge), rK2 (response K1), A2 (secondary actor). • Maintained the role of facilitator.	• Engaged in K2 (secondary knowledge), Q1 (confused response), X1 (inappropriate response). • Remained in the position as a novice.

evolved into an initiator, demonstrating self-efficacy, while NP1 remained passive.

These findings highlight that role shifts within a group can occur due to the roles and activities performed by members during discussions. Active engagement in activities, such as knowledge repetition and discussion control, influences positional changes within the group, enhancing shared understanding and individual contributions..

Group 2-Dynamic evaluators

Group 2 demonstrated that despite shifts in roles, the role structure within the group remained relatively stable. EP1, FP1, and NP1 started from the same positions as in group 1. However, EP1 underwent a significant transformation from facilitator to evaluator (F'P1), focusing on self-talk and error correction. FP1 continued to serve as the facilitator, while NP1 remained in the novice (NP1) position.

This change in position was driven by activities such as K1 (prior knowledge), dK1 (delayed primary knowledge), and K2 (secondary knowledge). EP1 transitioned from a response role to that of an evaluator, while FP1 stayed as a facilitator, contributing to the discussion, and NP1 remained passive. The shift of EP1 to evaluator underscores how group members adapt to changes in tasks and roles.

These findings illustrate that even though group members focus on commognitive components like word use (WU), visual mediators iconic (VMI), and routine corrigibility (RC), they still encountered difficulties in applying Polya's strategies. This highlights the challenges associated with employing problem-solving strategies and the importance of a deep understanding of commognitive components in collaborative contexts.

Relationship With Presented Literature

Group 1

The findings from group 1 are consistent with the literature suggesting that positioning in group discussions influences learning dynamics and effectiveness (DeJarnette & González, 2015). The shift from expert positions to facilitator and vice versa reflects findings that dynamic and collaborative interactions enhance individual understanding and contributions (Johnson et al., 2014; Staples, 2014). This aligns with views that active roles in group discussions facilitate collaborative learning (Azmitia, 2016; Yu & Hu, 2017).

Group 2

The findings from group 2 also align with the literature that emphasizes the importance of positioning in group interactions (Esmonde, 2013; Zhang et al., 2019). EP1's transition from facilitator to evaluator illustrates how positioning and self-talk impact the evaluation process and error correction. This supports findings indicating that communication and roles in group discussions are crucial in influencing problem-solving effectiveness (Campbell & Hodges, 2020).

Role analysis and commognitive components

The findings regarding the relationship between positioning, Polya's problem-solving strategies, and commognitive components underline the complexity of interactions in group discussions. The literature indicates that problem-solving involves deep cognitive processes and communication (Sfard, 2001). In group 1, role changes correlated with increased communication and collaboration skills, whereas in group 2, limited use of commognitive components highlighted challenges in effectively applying problem-solving strategies.

Relationship Between Position, Problem-Solving Strategies, and Commognitive Components

Position and problem-solving strategies

Positioning in group discussions affects the application of problem-solving strategies. In group 1, the shift from expert to facilitator indicates adaptation in response to group dynamics and problem-solving tasks (DeJarnette, 2018). This suggests that role flexibility influences how group members apply the steps of Polya's strategy and solve problems effectively.

Commognitive components

The use of commognitive components, such as word use (WU), visual mediators (VMS, VMI), and routines (RE, RC), demonstrates how communication and understanding evolve during group discussions. In group 1, activities like knowledge repetition and discussion control facilitated role shifts and enhanced member contributions (Caspi & Sfard, 2012; Kim et al., 2017). Conversely, in group 2, challenges in effectively using commognitive components contributed to more limited position shifts and difficulties in applying problem-solving strategies (Sfard, 2006; Viirman, 2015).

Interaction between commognitive components

The commognitive components interact in the problem-solving process. The use of visual mediators and narratives supports conceptual understanding and idea exploration, while corrective routines assist in error correction (Sfard et al., 1998). In group 1, the interaction among these components supported role adaptation and problem-solving, while in group 2, less effective use of these components indicates challenges in applying strategies and developing understanding.

Implications for collaborative learning

These findings highlight the importance of understanding how positions and roles in group discussions influence problem-solving strategies and the use of commognitive components. Role changes can facilitate or hinder the problem-solving process, depending on how group members interact and utilize commognitive components (Harré, 2015; Kayı-Aydar, 2019). Therefore, learning strategies that support role flexibility and effective communication in group discussions can enhance the effectiveness of mathematics learning and problem-solving.

CONCLUSION

Transformative Trio (Group 1)

The findings indicate that group members experienced significant role shifts during the problem-solving process. Members who were initially dominant (EP1) transformed into facilitators, while the initial facilitator (FP1) became more like an expert. This suggests that active engagement and adaptation in discussion activities, such as knowledge repetition and discussion control, can enhance individual contributions and shared understanding.

Dynamic Evaluators (Group 2):

Despite role shifts, the role structure within the group remained relatively stable. Members initially serving as facilitators (EP1) transitioned into evaluators focusing on self-talk and error correction. This shift indicates that although there were changes in tasks and roles, difficulties in applying Polya's strategies persisted, highlighting challenges in problem-solving and the importance of a deep understanding of commognitive components in collaborative contexts.

These findings align with literature suggesting that positioning in group discussions influences learning dynamics and effectiveness. Role changes highlight the importance of dynamic and collaborative interactions in enhancing individual understanding and contributions. Commognitive components such as word use, visual mediators, and corrective routines also influence the problem-solving process, underscoring the complexity of interactions in group discussions.

Implications for Theory and Practice

Implications for theory

1. **Positioning in groups:** The shifts in roles within groups indicate that the dynamics of positioning are crucial in the problem-solving process. Theories related to group interaction need to consider role flexibility and member adaptability.
2. **Commognitive components:** The usage of commognitive components impacts how group members communicate and understand problems. Theories on problem-solving should integrate a deep understanding of how these components interact during group discussions.

Implications for practice

1. **Designing collaborative activities:** Educators need to design activities that support role flexibility and active engagement of all group members. Activities that encourage knowledge repetition and discussion control can enhance individual understanding and contributions.
2. **Utilizing commognitive components:** Teachers should instruct students on effectively using commognitive components during group discussions. This can assist students in applying problem-solving strategies and improving their understanding of discussed concepts.

Recommendations for Future Research and Practical Applications

Future research directions

1. **Longitudinal studies:** Further research should be conducted to understand how role shifts occur over the long term and how this affects students' understanding and performance.
2. **Influence of commognitive components:** Further studies on how different commognitive components interact during the problem-solving process and how this affects student learning outcomes.
3. **Context variability:** Research in various educational and cultural contexts to see whether these findings are consistent or if there are significant variations.

By understanding and implementing these findings, both educational theory and practice can evolve to support more effective collaborative learning and enhance students' problem-solving skills.

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