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Student learning trajectories in finding the perimeter and area of a rectangular in the context of a fishing pond

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Abstract
Mathematics is a difficult subject for elementary school students. One of the causes is errors in mathematical concepts and principles made by students. For this reason, efforts need to be made to improve the concept and principles of quadrilaterals. The aim of this research is to design student learning trajectories in finding the perimeter and area of a rectangle in the context of a fishing pond. This is research with a design research approach. The subjects of this research were 50 students divided into two groups, 25 people each. Data collection techniques through observation, interviews and documentation. The data analysis technique used is qualitative and is developed through certain relationship patterns. The results of this research show that the mathematics learning trajectory design was implemented very well. It improves the ability of understanding about quadrilaterals. The learning path is activity-1: students create a representation of the context of a fishing pond and a rectangular image. Activity-2: students determine the perimeter of a rectangular image by placing match sticks around the rectangle in sequence, holding them tightly together. Activity-3: students determine the circumference of the fishing pond. Activity-4: students determine how to calculate the circumference of the pond by drawing a rectangle using matchsticks. Activity-5: students discover the principle of perimeter of a rectangle. The conclusion of this research is that the mathematics learning trajectory design is valid, practical, and increases the ability to understand concepts and principles from quadrilaterals.

Keywords: learning trajectory, quadrilateral, area, perimeter, middle school students

INTRODUCTION

Geometry is a system built from abstract concepts, namely the mathematics is a basic discipline that plays an important role in various aspects of our daily lives. It provides provisions for students to analyze, interpret, and solve various problems. From managing finances to understanding complex scientific phenomena, mathematics serves as a bridge connecting theoretical concepts with practical applications (Nugroho et al., 2022a). Mathematics allows us to understand the world around us by providing a language and framework for understanding patterns, relationships, and quantitative information. Through mathematical thinking, we can approach problems systematically and logically, breaking them down into manageable steps and finding efficient solutions. By utilizing mathematical reasoning, we can analyze data critically, identify patterns, and make informed decisions (Anggoro et al., 2023a). Furthermore, mathematics encourages the development of critical thinking skills, as it requires us to evaluate and make logical arguments based on evidence. Apart from that, mathematics is a basic science that plays an important role in various aspects of students' daily lives. It is a thinking tool for solving various problems. Starting from managing finances to understanding complex scientific phenomena, mathematics functions as a bridge that connects theoretical concepts with practical applications (Sukestiyarno et al., 2024).

In real world scenarios, understanding the concepts of perimeter and area of rectangular shapes becomes important. For example, in a construction project, calculating the perimeter and area of a rectangular plan helps determine the number of materials needed, such as

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Contribution to the literature

- This research finds local learning theories about the perimeter and area of rectangles in the context of fishing ponds.
- The findings show that there is a learning trajectory for students in understanding the concepts and principles of geometry, especially rectangles, through the fishing pond approach.
- The research results show that the design of the geometry learning trajectory is valid, practical, and increases the ability to understand concepts and principles of quadrilaterals.

flooring or paint (Anggoro et al., 2023b). In addition, in gardening or landscaping, knowing the perimeter and area of a rectangular plot of land helps in determining the number of fences or plants needed. The question of exploring the circumference when the area is known (Sukestiyarno et al., 2023). In addition, in the field of urban planning, knowledge of the perimeter and area of rectangular sites is very important for designing and optimizing land use (Widada et al., 2019a). In architecture and interior design, understanding the perimeter and area of a rectangular room is critical to optimizing the layout and determining furniture placement. In terms of financial planning or budgeting, knowing the perimeter and area of a rectangular room can help in calculating renovation costs or determining the rental or purchase price of a property (Anggoro et al., 2023b). Additionally, in agriculture, calculating the perimeter and area of rectangular land helps farmers determine the number of fences needed to secure livestock or crops, as well as assisting in irrigation planning and land management.

In this lesson, students will develop their ability to understand and identify various types of quadrilaterals. Students will be introduced to the concept of quadrilaterals and their characteristics through interactive activities and teaching aids (Widada et al., 2019c). These activities may include sorting shapes into quadrilaterals and non-quadrangles, identifying properties of quadrilaterals such as parallel sides, equal angles, and diagonals, and exploring real-life examples of quadrilaterals in architecture and nature. Through this activity, students not only deepen their understanding of quadrilaterals but also develop spatial awareness and geometry skills. By using digital and physical manipulatives, students will be able to visualize and manipulate quadrilaterals, thereby increasing their understanding of the topic (Lubis et al., 2021). Additionally, students will be encouraged to engage in problem-solving tasks that require them to apply their knowledge of quadrilaterals. At the end of this lesson, students are expected to be able to confidently recognize and differentiate various types of quadrilaterals based on their properties (Herawaty et al., 2020a, 2021a). The integration of digital and physical manipulatives in this learning aims to support the development of students' visualization skills and improve their understanding of quadrilaterals in a meaningful and interesting way (Agusdianita et al., 2021). The main idea of this research is to explore how digital and physical manipulatives can be integrated into solid geometry learning to support students' visualization development.

In this lesson students will develop their ability to understand and identify various types of four areas. Students will be introduced to the concept of quadrilaterals and their characteristics through interactive activities and teaching aids (Salmaini et al., 2021). These activities can include sorting shapes into quadrilaterals and non-quadrangles, identifying the properties of quadrilaterals such as parallel sides, equal angles, and diagonals, and exploring real-life examples of quadrilaterals in architecture and nature (Herawaty et al., 2021b). Through this activity, students not only deepen their understanding of quadrilaterals but also develop spatial awareness and geometry skills. By using digital and physical manipulatives, students will be able to visualize and manipulate rectangles, thereby increasing their understanding of the topic. Additionally, students will be encouraged to engage in problem-solving tasks that require them to apply their knowledge in four areas. At the end of this lesson, students are expected to be confident in identifying and differentiating various types of quadrilaterals based on their properties (Nuryadi et al., 2021; Widada et al., 2019c). This integration of digital and physical manipulatives in learning aims to support the development of students' visualization skills and improve their understanding of quadrilaterals in a meaningful and interesting way (Widada et al., 2019d).

On the other hand, students often have difficulty in understanding the concept of spatial geometry, especially in distinguishing between flat geometry and spatial geometry (Nugroho et al., 2022b). This is because spatial geometry involves three-dimensional objects that require higher spatial visualization skills (Nugroho et al., 2021; Sukestiyarno et al., 2023). This visualization ability allows students to visualize shapes and spaces in their minds. In addition, students also have difficulty understanding the relationships between shapes, such as how cones and cylinders can be connected (Widada et al., 2021). They may have difficulty imagining the pieces of a shape and understanding how its various parts are connected to each other. The main idea of this research is to explore how digital and physical manipulatives can be integrated into solid geometry learning to support students' visualization development. It is a process of mathematical thinking in an effort to reach a certain concept.

Mathematical thinking is an integral part of developing a deep understanding of mathematics. It involves a mindset that includes curiosity, creativity, and persistence in solving problems. Mathematical thinking is more than just memorizing formulas and algorithms (Widada et al., 2019b). It involves the ability to analyze and break down complex problems into small, manageable parts, recognize patterns and relationships, and apply logical reasoning to develop new strategies for problem solving (Herawaty et al., 2019a, 2019b). Key components of mathematical reasoning include analyzing information and evidence, making conjectures and forming hypotheses, creating and testing strategies, recognizing patterns and relationships, making logical deductions and reasoning, communicating mathematical ideas and solutions clearly, and reflecting on the problem-solving process (Roza et al., 2020). These components work together to improve our understanding of mathematical concepts, promote critical thinking skills, and enable us to solve problems effectively (Widada et al., 2020a, 2020b). By developing their mathematical thinking skills, students can become more confident and proficient in their ability to solve problems, both in mathematics and in other disciplines as well. By developing their mathematical thinking skills, students can become more confident and proficient in their ability to solve problems, both in mathematics and in other disciplines as well. Mathematical thinking is an important skill that goes beyond the classroom.

Developing a mathematics mindset involves adopting a positive attitude toward mathematics, embracing challenges, assessing mistakes as opportunities to learn, and persevering in the face of difficult problems (Hamami et al., 2020). It also involves being open-minded and willing to explore different strategies and approaches to problem solving. By developing a mathematical mindset, individuals can improve their ability to think critically and logically, analyze and solve problems, and make decisions in various aspects. The role of logic in mathematics is very important. Logic provides the basis for mathematical reasoning and proof (Nugroho et al., 2021). This allows mathematicians to construct valid arguments, make logical deductions, and ensure the accuracy and consistency of mathematical statements. Strategies to improve mathematical thinking include encouraging creative and exploratory thinking, providing opportunities for problem solving and hands-on learning, cultivating a growth mindset, emphasizing the process rather than just the results, promoting collaboration and discussion among students, and incorporating real-world mathematical applications (Ganal & Guiab, 2014). Some strategies that can be implemented to improve mathematical thinking include encouraging creative and exploratory thinking, providing opportunities for problem solving and hands-on learning, encouraging the development of mathematical thinking.

Abstract mathematical concepts can be difficult to understand for many people. But with patience and practice, it is possible to develop a deep understanding of these complex ideas (Jaenudin et al., 2020). One effective approach is to break abstract concepts into smaller, more concrete parts, and use examples and analogies to help make ideas more concrete. Another useful strategy is to tackle problems step by step, building simpler concepts and gradually adding complexity (Sukestiyarno et al., 2021). With persistence and a willingness to learn, anyone can develop a strong foundation in abstract mathematics (Herawaty et al., 2020b). Mathematical thinking can be very useful in solving real-world problems (Chen et al., 2021). By breaking down complex problems into smaller, more manageable parts, it is possible to identify patterns and relationships that may not be immediately apparent. This can be especially helpful when dealing with large data sets or when trying to make predictions based on incomplete information (Sukestiyarno et al., 2019). One effective strategy is to use mathematical modeling, which involves creating simple representations of real-world phenomena using mathematical equations. By manipulating these equations, it is possible to explore different scenarios and predict how the system will behave under different conditions. For example, mathematical modeling can be used to predict the spread of disease, the impact of climate change on ecosystems, or the behavior of financial markets.

Another useful approach is to use statistical analysis to identify trends and patterns in the data. This can involve calculating the mean, standard deviation, and other steps to better understand the underlying distribution of the data (Widada & Herawaty, 2023). By identifying outliers and other anomalies, it is possible to identify areas of concern and take action to address them. Ultimately, applying mathematical thinking to real-world problems requires creativity, persistence, and a willingness to experiment with different approaches (Pamungkas & Nugroho, 2020; Widada & Herawaty, 2023). But with practice, it is possible to develop a powerful toolkit for addressing complex problems and making informed decisions.

The intersection of mathematics and cognitive science is an exciting field that explores how mathematical concepts can be used to better understand the human mind. By applying mathematical models to cognitive processes such as perception, memory, and decision making, researchers gain new insights into how the brain works (Yuliardi & Rosjanuardi, 2021). For example, mathematical models can be used to study how people make decisions under uncertainty, how they learn from feedback, and how they integrate information from various sources (Bonasera & Bosanac, 2021). These models can help us understand why people make certain choices, and how they may be influenced by factors such as emotions, context, and social cues. To explore the characteristics of critical and creative thinking in mathematics education students, it is important to understand that critical thinking and creative thinking are interconnected (Changwong et al., 2018). Critical thinking lays the foundation for creative thinking, as it involves the ability to analyze and evaluate information, make logical deductions, and consider different perspectives. Creative thinking in mathematics involves generating new ideas and approaches to problem solving, while critical thinking ensures that these ideas are logical, valid, and well-founded (Suharto & Widada, 2019).

Creative thinking in mathematics is more than just finding solutions. This involves the ability to think outside the box, apply innovative strategies, and make connections between different mathematical concepts (Amri & Widada, 2019). By developing a mathematical mindset, individuals can improve their ability to think critically and logically, analyze and solve problems, and make informed decisions in various aspects of their lives. Developing creative thinking in mathematics is very important for students because it allows them to approach problems from different angles, think outside the box, and come up with unique and innovative solutions. By incorporating strategies to improve mathematical thinking, such as encouraging creative thinking and exploration, providing opportunities for problem solving and hands-on learning, fostering a growth mindset. As explained previously, mathematics is recognized as a fundamental and essential subject that forms the basis of various scientific disciplines (Zakeri et al., 2023). Therefore, students' learning trajectories are needed that are in accordance with their cognitive processes. Student learning trajectories need to be designed appropriately, to make it easier for students to achieve a mathematical concept or principle.

Designing mathematics learning trajectories is critical to guiding students on their mathematics journey and ensuring their understanding and mastery of key concepts and skills (Baroody et al., 2022; Maurer, 1991; Simon, 2020; Simon & Tzur, 2012). The trajectory must be appropriate to the student's characteristics, valid in content and approach, easy to understand, and ultimately contribute to the development of students' mathematical communication skills. By designing mathematics learning trajectories, educators can create a structured and coherent path for students to follow, allowing them to progressively build on previous knowledge and skills. In addition, the design of mathematics learning trajectories must also consider students' interests, engagement and identity formation (Zakeri et al., 2023). By combining engaging teaching principles, mathematics learning trajectories can be made more motivating, interactive, and effective in facilitating deep learning (Winangun & Fauziah, 2019). Designing a mathematics learning trajectory is very important to provide students with structured and coherent learning. pathways to develop their mathematical understanding and skills. This can be achieved by carefully sequencing and structuring content, ensuring that the necessary knowledge has been acquired before moving on to more advanced concepts (Mpofu, 2020). In addition, designing mathematics learning trajectories allows educators to meet diverse student needs and differentiate instruction accordingly (Winangun & Fauziah, 2019). This may include providing additional support or outreach activities, incorporating real-life applications and problem-solving tasks, and utilizing technology to enhance the learning experience (Zakeri et al., 2023). In addition, designing mathematics learning trajectories can create a more motivating and engaging classroom environment for teachers and students (Rahman et al., 2021). Overall, the design of mathematics learning trajectories is very important in encouraging effective education and developing students' twenty-first century competencies (Ivars et al., 2018; Simon et al., 2018; Winangun & Fauziah, 2019). By designing mathematics learning trajectories, educators can create a structured and coherent path for students to follow, allowing them to progressively build on previous knowledge and skills. Designing mathematics learning trajectories is critical to providing students with a structured and coherent path to developing their mathematical understanding and skills. This can be achieved by carefully sequencing and structuring content, ensuring that the necessary knowledge has been acquired before moving on to more advanced concepts (Mpofu, 2020).

Therefore, it is necessary to design student learning trajectories in understanding the concepts and principles of quadrilaterals using the context of everyday life. This makes it easier for students to carry out mathematical thinking processes in discovering and using quadrilateral concepts and principles. The problem of this research is how to design student learning trajectories in finding the perimeter and area of a rectangle in the context of a fishing pond?

METHOD

This research is validation research with a type of design research. Validation studies focus on designing hypothetical learning trajectories (HLT) to develop the learning process and describe the design process (Plomp & Nieveen, 2013). The research aims to produce HLT based on fishing tourism in a fishpond. This research involved 50 junior high school students in Riau, Indonesia. The research was conducted in three stages:

Table 1. Activities and HLT

Main activity	Main goal	Hypothetical activity
Activity-1: Students make a representation of the context of Mr. Roy's fishing pond and the shape of the image is rectangular.	Represents the context of Pak Roy's fishing pond and is a rectangular image with a length of 7 and a width of 4.	• Students make a rectangular drawing as a representation of Pak Roy's pond, which measures 7 meters long and 4 meters wide. • Students use the parable that the size of 1 meter in the picture is represented by 1 matchstick.
Activity-2: Students determine the perimeter of a rectangular image by placing matchsticks around the rectangle in sequence, holding them tightly together.	Place the match sticks around the rectangle in sequence, pressing them tightly together, to determine the perimeter of the rectangle.	• Students place match sticks around the rectangle in sequence and touch each other tightly until they return to the starting point. • Students determine the number of sticks placed around the rectangle, namely 22 sticks.
Activity-3: Students determine the circumference of the fishing pond.	Determine the perimeter of the rectangle to find the perimeter of the fishing pond.	• Students determine the perimeter of the rectangle is 22 units. • Students use the example that 1 matchstick is 1 meter, then students conclude that the circumference of Pak Roy's pond is 22 meters.
Activity-4: Students determine how to calculate the circumference of the pond by drawing a rectangle using matchsticks.	Create a way to calculate the circumference of the pond using a rectangular image using match sticks.	• Students calculate the number of sticks, namely 2 times 7 sticks + 4 sticks equals 2 x (7 + 4) = 22 • Students solve the pond perimeter problem by representing a rectangular image using match sticks.
Activity-5: Students discover the principle of perimeter of a rectangle.	Create a formula for the perimeter of a rectangle.	• Students relate the solution to the formula for the perimeter of a rectangle and provide logical arguments. • Students find the formula for the perimeter of a rectangle (K) by assuming the length is p and the width is l, the perimeter of the rectangle K = 2 x (p + l).

preparing the experiment, experimenting in class, and conducting retrospective analysis (Fauzan et al., 2002).

Preparation and Design

In the initial stage, the researcher prepares an initial design of an HLT. There are three activities in this stage. First, researchers gave students a pretest problem related to the area and perimeter of a quadrilateral. The pretest results show that students tend to have difficulty understanding the meaning of the problem, have difficulty understanding the problem/question, have difficulty using formulas, and there are students who have difficulty in the calculation process. Second, researchers conducted a literature review on various ways of teaching about the area and perimeter of quadrilaterals. Based on this study, it was found that teaching about the area and perimeter of quadrilaterals uses the broad context of painting walls, installing tiles on the floor, and fishing ponds. Third, the researcher chose the context of a fishing pond, and solved problems related to the area and perimeter of a rectangle to find concepts and principles.

Based on the results of this initial stage, the researcher developed an HLT consisting of three components: learning objectives, learning activities, and the hypothesized learning process (Plomp & Nieveen, 2013). This is stated in **Table 1**.

There are five criteria for selecting "Mr. Roy's fishpond" as the context in this study. The five criteria are

(1) its location is close to the school where this research was conducted,

(2) it is very familiar to students as research subjects,

(3) the pond is rectangular in shape,

(4) it is often used for fishing competition events for students, and

(5) the pond atmosphere is very cool which supports students learning while fishing.

Teaching Experiments

The teaching experiment stage was carried out as a pilot experiment. This was an initial trial of the HLT design which was carried out involving five students working in one group to obtain input for improving the HLT design. The input is about the readability of the worksheets, the clarity of instructions and commands, as well as the attractiveness and appropriateness of the context used in student learning. Researchers revised the HLT design to be implemented in a pilot experiment involving all students. Those are two classes of 25 students each. During the learning process, researchers collected data through learning observation videos, student answers on student worksheets, anecdotes, and interviews.

Figure 1. Pak Roy's fishing pond (Source: Authors' own elaboration)

Retrospective Analysis

At the retrospective analysis stage, researchers compared the implementation of learning, namely between the actual learning trajectory (ALT) and the HLT design. Also, researchers identified how students found, understood and used the context of fishing ponds. Data obtained from videos, field notes, and interviews were analyzed to find fixed moments that illustrate students' first understanding of quadrilateral concepts and principles. The findings at this stage are used to explain the role of the fishing pond context in helping students discover and understand quadrilateral concepts and principles. Data regarding the validity and reliability of HLT components (per sub concept) were analyzed using the Lisrel 8.8 application program.

RESULTS

Context of Pond Problems for Fishing Tourism

On Sundays, Pak Roy's fishing pond is packed with visitors. Pak Roy's hut is rectangular in shape with a width of 4 meters and a length of 7 meters (see **Figure 1**). To ensure the capacity of visitors who could enter the pond, Pak Roy measured the circumference and area of the pond. How can you find it, if you are asked to help Pak Roy determine the circumference and area of the pond?

Analysis of interview excerpts between the researcher (R) and the research subject (Bt) is, as follows.

R: How do you explain the perimeter of a rectangle?

Bt.01: I tried to make a picture of Pak Roy's pond which is 4 meters wide and 7 meters long. I use the example of 1 meter in the picture being represented by 1 matchstick.

R: OK... what next?

Bt.02: Can you draw a picture of what I have made [meaning see **Figure 2**] ... starting from the very beginning, as in the picture, I placed matchsticks around the rectangle in sequence and pressed them together tightly up to the starting point.

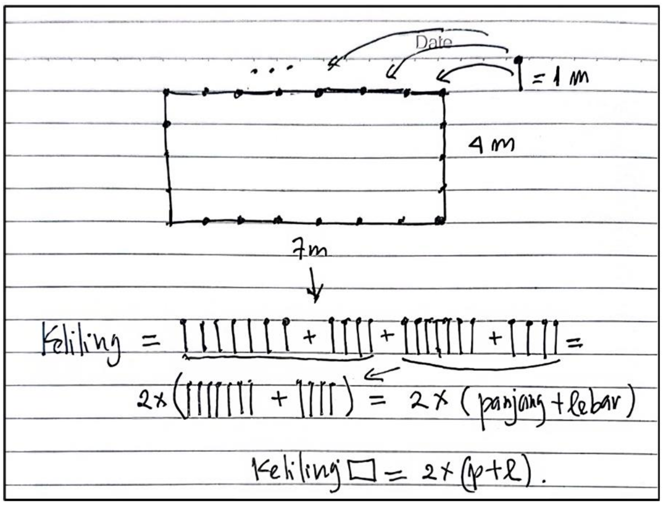

Figure 2. Discovery of the perimeter of a rectangle by Bt (Source: Authors' own elaboration)

R: OK ... how did it go?

Bt.03: ... it turns out the number of matchsticks was 22. That means that the perimeter of the rectangle is 22, and because 1 matchstick represents 1 meter, I can conclude that the circumference of Pak Roy's pond is 22 meters.

R: How did you get those 22?

Bt.04: I can calculate as in my working paper [**Figure 2**], namely 7 sticks + 4 sticks + 7 sticks + 4 sticks is the same as 2 times 7 sticks + 4 sticks or written 2 x (7 + 4) = 22.

R: What is your conclusion?

Bt.05: Because the length is 7 and the width is 4, and I let the length be p and the width be l, then the perimeter of the rectangle K = 2 x (p + l).

The rest of the interview is:

R: OK ... now how do you determine the area of Pak Roy's pond?

Bt.06: As I described Pak Roy's pond in my previous work, make a unit square with the size of 1x1 matchstick which represents 1 m^2. I made as many unit squares as possible [See **Figure 3**].

R: What next?

Bt.07: I started from the base, I covered the rectangle with unit squares side by side until everything was completely covered without any more gaps.

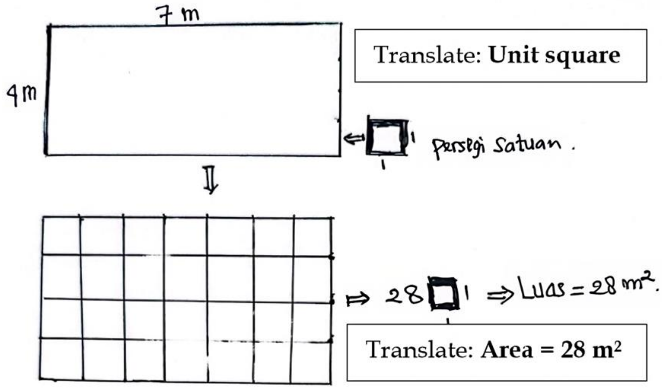

Figure 3. Discovery of the area of a rectangle by Bt (Source: Authors' own elaboration)

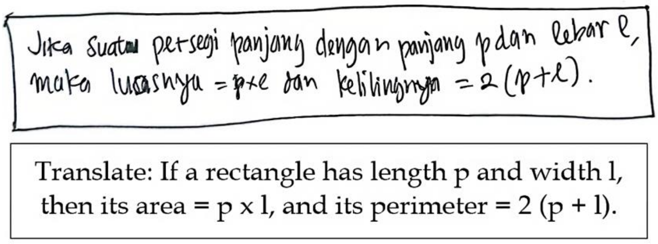

Figure 4. Conclusion of Bt, area, and perimeter of the rectangle (Source: Authors' own elaboration)

R: OK ...

Bt.08: Then I counted the number of unit squares, the result was 28 unit squares. That means that the area of Pak Roy's pond is 28 m^2.

R: Why 28 m^2?

Bt.09: Yes sir, ... that's because 1 unit square represents 1 m^2, so 28 unit squares = 28 m^2.

R: What is the relationship between 28 m^2 and the length and width of Pak Roy's pond?

Bt.10: Because the length is 7 m and the width is 4 m, the area of Pak Roy's pond is 7 m x 4 m = 28 m^2.

R: Given a rectangle of length p and width l, how would you deduce the perimeter and area of the rectangle?

Bt.11: I have made a conclusion on my worksheet [Figure 4] is that the area of a rectangle = p x l, and its perimeter is 2(p + l).

Based on analysis of interviews with research subjects (BT students), students' ALT in solving contextual problems and achieving quadrilateral concepts and principles are, as follows.

Activity-1: Students make a representation of the context of Mr. Roy's fishing pond and the shape of the image is rectangular. Representing the context of Pak Roy's fishing pond and the shape of the image is a rectangle with a length of 7 and a width of 4. Students make a rectangular image as a representation of Pak Roy's pond which measures 7 meters long and 4 meters wide. Students use the parable that the size of 1 meter in the picture is represented by 1 matchstick. Meanwhile, for area, students use a unit square.

Activity-2: Students determine the perimeter of a rectangular image by placing match sticks around the rectangle in sequence, holding them tightly together. Place the match sticks around the rectangle in sequence, pressing them tightly together, to determine the perimeter of the rectangle. Students place match sticks around the rectangle in sequence and touch each other tightly until they return to the starting point. Students determine the number of sticks placed around the rectangle, namely 22 sticks. Meanwhile, for area, students determine the number of unit squares that cover the figure four without remaining.

Activity-3: Students determine the circumference of the fishing pond. Determine the perimeter of the rectangle to find the perimeter of the fishing pond. Students determine the perimeter of the rectangle is 22 units. Students use the example that 1 matchstick is 1 meter, then students conclude that the circumference of Mr. Roy's pond is 22 meters. For area, students find that the number of unit squares is 28. That is what they call the area of the rectangle.

Activity-4: Students determine how to calculate the circumference of the pond by drawing a rectangle using matchsticks. Create a way to calculate the circumference of the pond using a rectangular image using match sticks. Students calculate the number of sticks, namely 2 times 7 sticks + 4 sticks equals 2 x (7 + 4) = 22. Students solve the pond perimeter problem by representing a rectangular image using match sticks. As for the area, students can find that 28 is obtained from multiplying 4 and 7. This means that the area of the rectangular pond = 4 x 7 = 28 square meters.

Activity-5: Students discover the principle of perimeter of a rectangle. Create a formula for the perimeter of a rectangle. Students relate the solution to the formula for the perimeter of a rectangle and provide logical arguments. Students find the formula for the perimeter of a rectangle (K) by assuming the length is p and the width is l, the perimeter of the rectangle K = 2 x (p + l). Meanwhile, for rectangles, students conclude that if a rectangle has width l and length p, then its area is A = p x l. These student activities are in a very pleasant atmosphere as depicted in Figure 5.

To determine the level of implementation of the mathematics learning trajectory design, observations were made during the learning process. Data analysis resulting from observations of the implementation of the learning pathway design can be seen in Figure 6.

Based on Figure 6, the implementation of the mathematics learning trajectory design is very good.

Figure 5. Student learning atmosphere in class (Source: Authors' own elaboration)

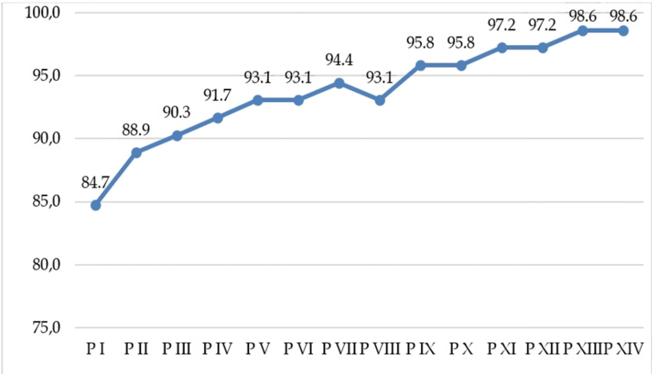

Figure 6. Graph of rectangular material implementation (Pi: Meeting i with i = I-XIV) (Source: Authors' own elaboration)

This is shown by the average implementation being 93.20%. The percentage of implementation is always above 84.50%, and continues to increase, starting from meeting I at 84.70% and increasing to meeting XIV with an implementation rate of 98.60%. This shows that the design of mathematics learning trajectories can improve understanding of concepts and principles about quadrilaterals very well.

Based on **Figure 5**, the implementation of the mathematics learning trajectory design is very good. This is shown by the average implementation being 93.20%. The percentage of implementation is always above 84.50%, and continues to increase, starting from meeting I at 84.70% and increasing to meeting XIV with an implementation rate of 98.60%. This shows that the design of mathematics learning trajectories can improve mathematics literacy activities very well.

Based on the implementation data, the validity and reliability of each *sub concept* of rectangular material can be analyzed. There are ten *sub concept* of rectangular material, whose adequacy is assessed by 50 students who take part in Rectangular material learning. Analysis using Lisrel 8.8 with the results depicted in **Figure 7** and **Figure 8**.

Then the output of Lisrel 8.8 to analyze the implementation of learning Rectangle material using the HLT that has been designed, a T-Score diagram is obtained (see **Figure 7**).

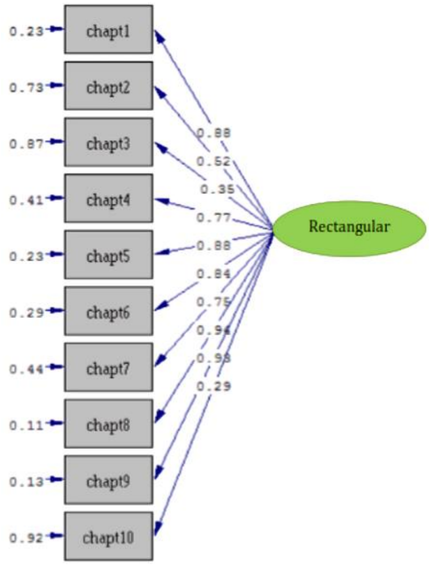

Figure 7. Standard solution of the basic model of learning implementation (Chi-Square=46.77, df=25, p-value=0.522, RMSEA=0.004) (Source: Authors' own elaboration)

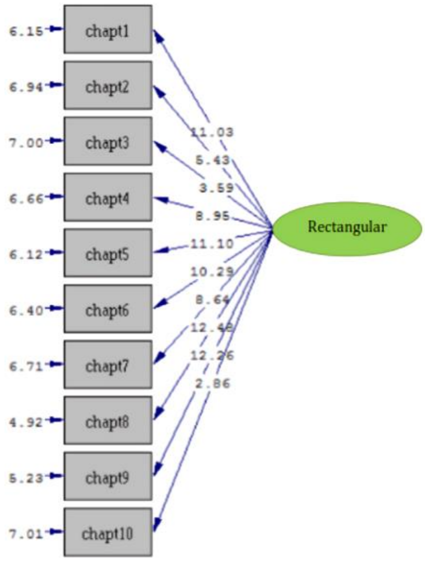

Figure 8. T-score basic model of learning implementation (Chi-Square=46.77, df=25, p-value=0.522, RMSEA=0.004) (Source: Authors' own elaboration)

Based on **Figure 7** and **Figure 8**, it can be analyzed the validity and reliability of the implementation of learning, as shown in **Table 2**.

Based on **Table 2** it can be described, as follows. Each sub concept (sub concept I, sub concept II, sub concept III, up to sub concept IX) of Rectangular material learning is validly implemented, even though there is an SLF that is less than 0.50, the t-score for each sub concept is > 1.96. In addition, the Learning CR index is 0.73 > 0.70. Thus it can be concluded that the implementation of Rectangular learning using HLT that has been designed is valid and reliable. This shows that learning has a complete combination of approaches, namely 10 sub-concepts (sub concept I-sub concept X).

Table 2. Validity and reliability of rectangular material learning implementation

Indicator	Standardized loading factors ≥ 0.50	Standard errors	t-value > 1.96	Declaration	Reliability (CR ≥ 0.70)
Chapt1	0.23	0.88	4.61	Good validity	
Chapt2	0.73	0.52	6.37	Good validity	
Chapt3	0.87	0.35	5.63	Good validity	
Chapt4	0.41	0.77	5.65	Good validity	
Chapt5	0.23	0.88	4.64	Good validity	0.73
Chapt6	0.29	0.84	3.75	Good validity	
Chapt7	0.44	0.75	4.81	Good validity	
Chapt8	0.11	0.94	3.87	Good validity	
Chapt9	0.13	0.98	3.47	Good validity	
Chapt10	0.92	0.29	4.05	Good validity	

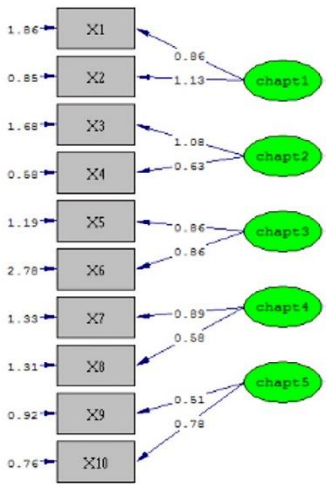

Figure 9. HLT implementation standard solutions (*sub concept* 1-5) (Chi-Square=3.94, df=19, p-value=0.243, RMSEA=0.043) (Source: Authors' own elaboration)

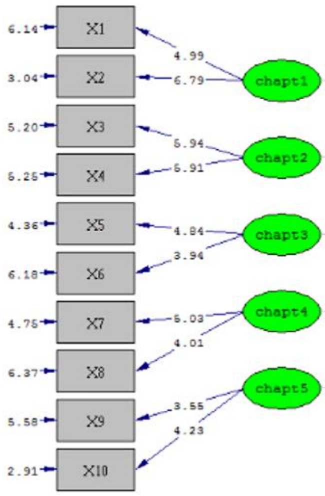

Figure 10. T-score of HLT implementation (*sub concept* 1-5) (Chi-Square=3.94, df=19, p-value=0.243, RMSEA=0.043) (Source: Authors' own elaboration)

Table 3. Validity and reliability of rectangular material learning implementation (*sub concept* 1-5)

Indicator	Standardized loading factors ≥ 0.50	Standard errors	t-value > 1.96	Declaration	Reliability (CR ≥ 0.70)
X1	0.86	1.86	4.99	Good validity	
X2	1.13	0.86	6.79	Good validity	
X3	1.08	1.68	5.94	Good validity	
X4	0.63	0.58	5.91	Good validity	
X5	0.86	1.19	4.84	Good validity	0.84
X6	0.86	2.78	3.94	Good validity	
X7	0.89	1.33	5.03	Good validity	
X8	0.58	1.31	4.01	Good validity	
X9	0.61	0.92	3.65	Good validity	
X10	0.78	0.76	4.23	Good validity	

Based on observations by two observers, the results of the trial analysis of the application of rectangle material learning by applying the HLT that have been designed can be further explained. Analysis of observation data on the implementation of HLT using the help of the Lisrel 8.8 program. The results of the analysis are in the form of a CFA Part I diagram, as shown in **Figure 9** and **Figure 10**.

While the output of the basic model T-value from the observation data of the implementation of rectangular material learning (Subconcept 1-5) is, as follows: based on **Figure 9** and **Figure 10**, validity and reliability (CR) can be determined as in **Table 3**.

Based on **Table 3** it can be determined that each indicator X1-X10 has an SLF > 0.05 and a T-score > 1.96. It shows that the validity of each indicator from X1 to X10 is good validity. In the last column of **Table 3** it is found that CR = 0.84 > 0.70 which means that the implementation of HLT for rectangular teaching materials is reliable.

Next, the output of the CFA diagram of rectangular material learning implementation (Subconcept 6-10) is presented, as shown in **Figure 6** and **Figure 7**.

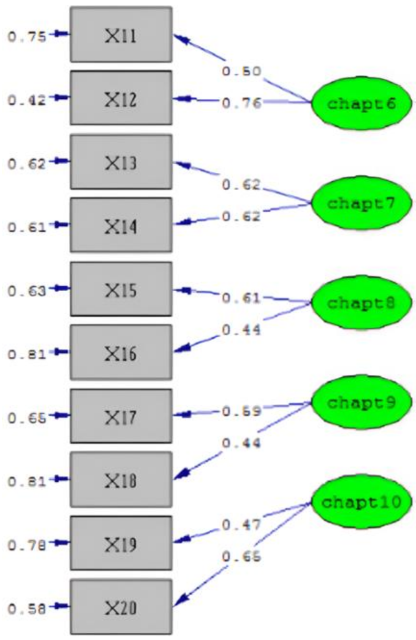

Figure 11. HLT implementation standard solutions (*sub concept* 6-10) (Chi-Square=63.43, df=34, p-value=0.279, RMSEA=0.079) (Source: Authors' own elaboration)

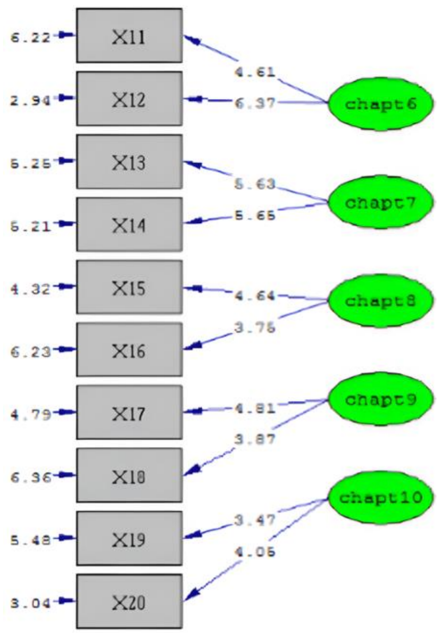

Figure 12. T-score of HLT implementation (*sub concept* 6-10) (Chi-Square=63.43, df=34, p-value=0.279, RMSEA=0.079) (Source: Authors' own elaboration)

Table 4. Validity and reliability of rectangular material learning implementation

Indicator	Standardized loading factors ≥ 0.50	Standard errors	t-value > 1.96	Declaration	Reliability (CR ≥ 0.70)
X11	0.50	0.75	4.61	Good validity	
X12	0.76	0.42	6.37	Good validity	
X13	0.62	0.62	5.63	Good validity	
X14	0.62	0.61	5.65	Good validity	
X15	0.61	0.63	4.64	Good validity	0.83
X16	0.44	0.81	3.75	Good validity	
X17	0.59	0.66	4.81	Good validity	
X18	0.44	0.81	3.87	Good validity	
X19	0.47	0.78	3.47	Good validity	
X20	0.66	0.58	4.05	Good validity	

Based on **Figure 11** is the standard solution output diagram for the basic model of sub concept 6- sub concept 10. This is the result of observing the implementation of learning rectangular material by applying HLT design. Meanwhile, the basic model T value output is, as follows: based on **Figure 11** and **Figure 12**, we can present **Table 4**. **Table 4** shows the validity and reliability of each sub-concept understood by students. Now look at **Table 4**.

Based on **Table 4** it can be determined that each indicator X11-X20 has an SLF > 0.05 except X16, X18 and X19, but all X10-X20 have a T-score > 1.96. It shows that the validity of each indicator starting from X11 to X20 is good validity. In the last column of **Table 4** it is found that CR = 0.83 > 0.70 which means that the Implementation of rectangular material learning is reliable. Thus, it can be concluded that all indicators of the Implementation of rectangular material learning from X1 to X20 are valid and the implementation of learning rectangular material learning (Subconcept 1-10) is reliable.

Based on the suitability analysis between ALT and HLT, it was found that ALT is in accordance with HTL, this provides evidence that the learning trajectory designed in this research is able to increase the ability to understand mathematical concepts and principles, especially quadrilateral material. The learning trajectory has five steps. That is activity-1: students create a representation of the context of a fishing pond and a rectangular image. Activity-2: students determine the perimeter of a rectangular image by placing matchsticks around the rectangle in sequence, holding them tightly together. Activity-3: students determine the circumference of the fishing pond. Activity-4: students determine how to calculate the circumference of the pond by drawing a rectangle using matchsticks. Activity-5: students discover the principle of perimeter of a rectangle.

DISCUSSION

The results of this research show that the real world context makes a positive contribution to increasing students' ability to understand mathematical concepts and principles. That is implementing an appropriately designed learning trajectory. In the field of education, understanding how students learn and the factors that influence their learning trajectories is very important (Apriyanti et al., 2019; Zhang & Wong, 2021). In learning trajectory studies, researchers aim to identify and analyze actual patterns and learning progress that students follow from time to time (Fuadiah, 2017). HLT, on the other hand, are theoretical models or frameworks that propose possible learning paths based on various factors and assumptions. This research suggests that HLT may indicate that students with high prior knowledge in a particular subject are most likely to make progress. Students more quickly and reach a higher level of understanding than students with low prior knowledge. Understanding students' true learning trajectories is critical to designing effective educational interventions and personalized learning experiences (Patsiomitou, 2018). In other words, although HLT can provide valuable insights and inform learning practices, it is important to base our understanding on ALT observed in real-world and classroom contexts (Panorkou & Greenstein, 2015; Salsabila et al., 2022). When studying ALT, researchers analyze real data and observe the learning patterns and progress that students follow over time (Setiadi et al., 2019). By examining students' actual learning sessions and behavior, researchers can gain insight into the factors that contribute to their success or struggle in a course.

The learning trajectory in math has proven to be effective in improving students' math abilities (Nasayao & Goyena, 2020). It provides a structured and sequential approach to teaching mathematical concepts and skills, allowing students to build upon their prior knowledge and develop a deeper understanding of mathematical principles. By following a learning trajectory in math, students are guided through a series of lessons and activities that are carefully designed to scaffold their learning. The approach helps students to make connections between different mathematical concepts, apply their knowledge to real-world problems, and develop problem-solving skills. Additionally, a learning trajectory in math allows educators to track students' progress and identify areas where further intervention may be needed (Jiménez-Fernández, 2016). Overall, the implementation of a learning trajectory in math has been shown to be instrumental in improving students' math abilities and facilitating their overall mathematical development (Gruver, 2018).Using the learning trajectory in math, students can improve their math abilities by following a structured and sequential approach that builds upon prior knowledge, develops deep understanding of mathematical principles, and enhances problem-solving skills (Jiménez-Fernández, 2016). The use of a learning trajectory in math has been shown to improve students' math abilities by providing a structured and sequential approach that builds upon prior knowledge.

HLT are theoretical models for the design of mathematics instruction (Simon, 2020). HLT consists of student learning goals, mathematical tasks that will be used to enhance student learning, and hypotheses about student learning processes (Simon & Tzur, 2012). Research on learning through activity has a mission to understand the learning of mathematical concepts in a way that can provide a strong foundation for instructional design (Simon et al., 2018). Thus, it is to provide prospective teachers with a specific language to describe student thinking (Ivars et al., 2018). Also, it is to rigorously evaluate the efficacy of using learning trajectories as a curricular and pedagogical tool and the key assumptions that underlie HLT-based instruction (Baroody et al., 2022). It has been demonstrated through several experiments on various mathematics topics and age groups (Simon, 2020).

Thus, we conclude that the mathematics learning trajectory design is valid, practical, and improves the ability to understand the concepts and principles of quadrilaterals. This learning trajectory has a syntax, namely five hierarchical activities. It provides empirical evidence that learning mathematics using real world contexts can make it easier for students to carry out the horizontal mathematization process towards formal mathematics.

CONCLUSION

This research concludes that the design of a learning trajectory about quadrilaterals using the context of a fishing pond was carried out very well. It improves the ability of understanding about quadrilaterals. The learning path is activity-1: students create a representation of the context of a fishing pond and a rectangular image. Activity-2: students determine the perimeter of a rectangular image by placing match sticks around the rectangle in sequence, holding them tightly together. Activity-3: students determine the circumference of the fishing pond. Activity-4: students determine how to calculate the circumference of the pond by drawing a rectangle using matchsticks. Activity-5: students discover the principle of perimeter of a rectangle. This mathematics learning trajectory is valid, practical, and increases the ability to understand the concepts and principles of quadrilaterals.

Based on the conclusion, it is suggested for teachers, prospective teachers, and mathematics learning designers to use the local cultural context in compiling geometry teaching plans and materials more broadly. That is in various contents and various local cultural contexts. That is the HLT design that can improve students' ability to understand geometry concepts specifically and mathematics in general.

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Ethical statement: The authors stated that the study was carried out in accordance with the guidelines of the Declaration of Helsinki and was approved by the Ethics Committee of the Mathematics Education Doctoral Program, Semarang State University, Indonesia on 10 September 2023 with code number: S-3Pmat-10.09.2003. Written informed consents were obtained from the participants.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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