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Effective teaching, teacher-student relation, student engagement and student mathematical achievement

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Abstract

This study tries to investigate the effects of effective teaching of mathematics on teacher-student relationship (TSR), student engagement (StEn) and student mathematical achievement. A questionnaire is developed to measure the above three constructs in this study, and examination results at the end of the academic year are obtained as the measure of achievement. Confirmatory factor analysis shows that the constructs measured by the questionnaire have reliability and validity. Structural equation modelling confirms the proposal that effective teaching will affect TSRs, StEn, and mathematical examination scores. Teaching effectiveness as measured by students' responses to the questionnaire can account for 14% of the variance of students' final examination score. This study may shed light on the causal effects of effective teaching on TSR, StEn and mathematical achievement. Findings of this study may inform teachers of approaches in teaching to enhance students' achievement in mathematics.

Keywords: effective teaching, student engagement, student mathematical achievement, teacher-student relationship

INTRODUCTION

Student achievement in examination is one of the key performance indicators of the quality of education provided by a school. In Hong Kong, student achievement of secondary school students is always treated as the most important indicator of school effectiveness by the public. As a result, teachers attach great importance to enhancing student examination results in secondary schools. Identifying factors for improving students' examination results will be seen as a top priority for teachers. There are many factors found to be significantly related to student achievement by research studies such as teaching effectiveness (TE) (Ding & Sherman, 2006; Odden et al., 2004), teacher-student relationship (TSR) (Allen et al., 2013; Cornelius-White, 2007; Roorda et al., 2011), and student engagement (StEn) (Alrashidi et al., 2016; Gunuc, 2014; Lei et al., 2018; Wang et al., 2016). This study tries to find out whether effective teaching of mathematics teachers can have positive effects on student achievement in mathematics through intervening variables of TSR and StEn.

LITERATURE REVIEW

Teaching Effectiveness

TE is understood and accepted by all stakeholders as a direct and important cause affecting student learning which in turn affects student achievement. The relationship is supported by many research studies. There are many research reports showing that TE has a direct relationship with student learning (Akram, 2019).

TE is a concept with different and diverse interpretations and measurements (Seidel & Shavelson, 2007). In order to understand what constitutes TE, different models are put forward to explain the teaching and learning process.

Stronge et al. (2011) suggested that there are four dimensions of TE according to a review of research studies. The four dimensions include instructional delivery, student assessment, learning environment and personal qualities. The instructional delivery can also be subdivided into instructional differentiation (Ryder et al., 2003), instructional focus on learning (Westley, 2011),

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Contribution to the literature

- This study found that teaching effectiveness, teacher-student relations and student engagement have causal effects on students' achievement as supported by the structural equation model analysis and the temporal differences in the measurement, without using an experimental design methodology.
- Engagement is found to have the highest impact on student mathematical achievement. In order to enhance students' mathematical achievement, great importance should be attached to improving student engagement during lessons.
- Teaching effectiveness has a higher positive influence on student engagement as compared to teacher-student relations. Teachers are advised to put more emphasis on improving their teaching effectiveness.

instructional complexity (Sternberg, 2003; Wenglinsky, 2000), expectations for student learning (Palardy & Rumberger, 2008; Peart & Campbell, 1999), use of technology (Cradler et al., 2002; Schacter, 1999), and questioning (Allington & McGill-Franzen, 2000). Stronge et al. (2011) found that there is a significant difference in student achievement in mathematics and reading, and classroom management between effective and less effective teachers.

A review of three international systems for observation of effective teaching, ISTOF, QoT and METE, supports that effective teaching with different dimensions can be observed. Instruction is one key dimension to be observed for effective teaching for all three systems. Some of the components of instruction include clarity and skills, enhancing students' conceptual development, activating prior knowledge to develop new concepts, adapting to students' characteristics, ability and needs.

Brophy (1986b) pointed out that effective teaching behavior is essential for improving student achievement. Muijs and Reynolds (2010) found that teacher behaviors were able to explain between 60% and 100% of pupils' progress on the numeracy tests in the United Kingdom. Findings by Kemp and Hall (1992) indicated that student achievement is linked to teacher competence, lesson presentation, teacher questioning techniques, discipline, and effective patterns of instruction. In a wide range of contexts and countries, effective teachers were found to emphasize academic instruction as their main classroom goal, have an academic orientation, create task-oriented environment, and spend classroom time on academic activities (Borich, 1996; Brophy, 1986a; Griffin & Barnes, 1986; Muijs & Reynolds, 2001).

There is also research mainly investigating the relationship between TE and student achievement (Ding & Sherman, 2006). For research on this relationship, TE will be more related to specific teaching practices (Kemp & Hall, 1992; Kyriakides, 2005), systematic teaching procedures (Kemp & Hall, 1992), and teaching connected to assessment (Porter, 2002).

It is proposed in this study that the instructional behavior of mathematics teachers in the classroom, as an important dimension of TE, will affect student achievement in mathematics.

Teacher-Student Relationship

TSR is also one important factor related to student achievement (Hamre & Pianta, 2001; Hughes et al., 2012; Ma et al., 2022; Midgley et al., 1989; Pianta & Allen, 2008; Xu & Qi, 2019). High school students with positive TSR have higher student mathematics achievement (Muller et al., 1999; Xu & Qi, 2019). Studies in China also found a positive association of TSR with students' mathematics achievement (Lei et al., 2022).

Roorda et al. (2011) adopted a meta-analytic approach to investigate the associations between affective qualities of TSR, students' school engagement and achievement and found that there are significant relations between TSR and students' school engagement and achievement. Roorda et al. (2011) concluded that the results provided further support to various prior research studies suggesting the influence of TSRs on students' school engagement and achievement (e.g., Bergin & Bergin, 2009; Davis, 2003; Hamre & Pianta, 2001; Pianta et al., 2003; Skinner & Belmont, 1993). Roorda et al. (2017) conducted a meta-analytic update and tested the mediating role of engagement between affective TSRs and students' achievement and concluded that affective TSRs will have positive effect on StEn which in turn affects students' achievement.

Student Engagement

Fredricks et al. (2019) pointed out that StEn is a prerequisite for student learning and retention. StEn is found to have significant effects on enhancing students' achievement (Center for Education Statistics and Evaluation [NSW], 2017). Lei et al. (2018) analyzed data from 69 independent samples to determine the relationship of StEn and academic achievement and found that there was a medium positive correlation of all facets of StEn with academic achievement. Gunuc (2014) found that there was a significant relationship between academic achievement and StEn.

Relationships Between TE, TSR, StEn, and Student Achievement

There are studies showing that TE (Allen et al., 2013; Cornelius-White, 2007); and StEn (Lei et al., 2018; Pedler et al., 2020) can affect student achievement (Lei et al.,

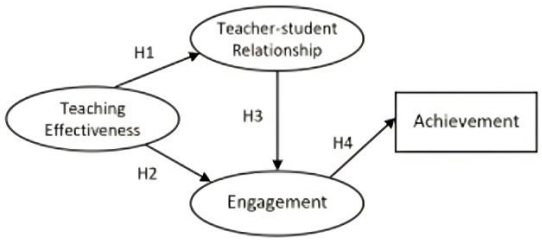

Figure 1. Theoretical model (Source: Authors' own elaboration)

2022; Roorda et al., 2011). There are also studies suggesting that TSR will affect StEn which in turn will affect student achievement (Hughes et al., 2008; Roorda et al., 2011).

Many studies found that TSR is significantly associated with student achievement (Emslander et al., 2023) and TE (Ghasemi, 2022; Li et al., 2022). Very few studies have investigated the casual effects between TSR and TE. Positive TSR was found to be a factor leading to TE (Li et al., 2022; Park & Kim, 2019), whereas Ghasemi (2022) found that TE might lead to TSR. The direction of causation between TE and TSR needs further research to confirm according to the above different findings.

One dimension of good quality of teaching in the QoT (Ingram et al., 2018), safe and stimulating learning climate, includes indicators of a relaxed atmosphere, mutual support, supporting the self-confidence of students, and showing respect for the students in behavior and language use. The indicators are conducive to developing a good TSR. The dimension of classroom climate in ISTOF (Ingram et al., 2018) for observing TE which includes teachers' warmth, empathy, warmth and respect to students, will very likely lead to a better TSR. The above dimensions of TE are likely to develop better TSR. Hence it is proposed in this study that TE may lead to TSR.

Theoretical Framework

This study tries to find out the relationship among TE of mathematics teachers, TSR, StEn, and students' performance in mathematics in a secondary school which admits students mainly from the below average ability group as measured in the secondary school allocation system. In order to find out the relationships among the constructs, a theoretical framework with reference to literature reviewed is put forward (**Figure 1).**

After having reviewed relevant literature, it is proposed that TE has causal effects on TSR (Ghasemi, 2022) as hypothesis 1; TE has positive effect on StEn as hypothesis 2; TSR has positive effect on StEn as hypothesis 3; and StEn has positive effect on student achievement as hypothesis 4.

TE is a multi-dimensional concept and in this study, it is proposed that the instructional behavior of mathematics teachers in the classroom, as an important dimension of TE, will affect students' achievement. In order to measure the instructional behavior of teachers, the method of using questionnaire designed for students is adopted. This method has been proved to be a possible, valid and reliable way of measurement (De Jong & Westerhof, 2001; Kyriakides, 2005; Kyriakides et al., 2014). As effective learning by students is the main goal of instructional behavior of teachers, students' perception of teacher's instructional effects should be a very appropriate indicator or measurement of teacher's instructional effectiveness. Students are the best people to judge whether they can learn effectively, especially in mathematics. Therefore, in this study, it is proposed that TE can be measured by teachers' behavior in the classroom related to the following aspects as perceived by students.

TE affects student achievement through some mediating variables. TSR and StEn are proposed to be two important mediating variables. There are studies showing that TSR (Allen et al., 2013; Cornelius-White, 2007); and StEn can affect student achievement (Lei et al., 2022; Roorda et al., 2011). There are also studies suggesting that TSR will affect StEn which in turn will affect student achievement (Hughes et al., 2008; Roorda et al., 2011). According to self-determination theory, there are three basic psychological needs of students for motivation and well-being: autonomy, competence and relatedness. In order to enhance students' learning effectiveness, teachers will adopt strategies in teaching to meet students' needs. The need for relatedness of students can be met by building a good TSR. Active engagement in learning will contribute to students' sense of learning with competence and autonomy. Hence, an effective teaching strategy which targets motivating students will probably lead to better teacher-student relations and students' engagement in learning. When the needs of students are met through good TSR and active engagement, their achievement in learning will be enhanced as a final result.

Samuelsson and Samuelsson (2017) found that an effective mathematics teacher worked patiently to establish structures, and there was almost no disruptive behavior because students simply were so engaged in learning mathematics. Understanding and building up concepts are important elements for learning Mathematics. If an effective teacher can explain concepts clearly to students, allow students to have sufficient time in thinking and understanding the concepts taught in the classroom with appropriate pace of teaching and learning activities so that students feel empowered to solve mathematical problems, students will be engaged in their learning. The above study may provide support to the proposal of effective teaching leading to StEn.

The above related studies suggest that effective teaching may have a causal effect on improving TSRs and StEn. There are also studies suggesting that TSR will

affect StEn which in turn will affect student achievement (Hughes et al., 2008; Roorda et al., 2011).

METHODOLOGY

Hypotheses

There are four hypotheses in this study according to the proposed theoretical framework as follows:

Hypothesis 1. Effective teaching has positive effects on TSR.

Hypothesis 2. Effective teaching has positive effects on StEn.

Hypothesis 3. TSR has positive effects on StEn.

Hypothesis 4. StEn has positive effects on student achievement.

Research Design

All form 4 students at a secondary school in Hong Kong were asked to respond to a questionnaire to measure the teacher's teaching strategy, TSR and StEn in the middle of an academic year. All items in the questionnaire adopt a 5-point Likert scale ranging from strongly disagree to strongly agree. There are 98 form 4 students studying in 4 classes taught by 4 different teachers, respectively. Students' examination scores in mathematics were collected at the end of the academic year.

Measures

Effective teaching. There are various dimensions related to TE proposed in various studies as reviewed above. This study will focus on instructional effectiveness in the classroom as students perceive it. There are research studies supporting that students' rating of teacher's effectiveness is reliable and valid (Akram, 2019; Kyriakides, 2005). This study will focus on instructional effectiveness in the classroom as perceived by students. Hence, TE is operationally defined as instructional effectiveness of teacher. With reference to indicators of TE (Ingram et al., 2018), the following characteristics of instructional effectiveness are measured with a questionnaire developed for this study:

1. Clarity of communication
2. Good instructional skills
3. Good pace of teaching process to allow students' thinking and conceptual building
4. Sufficient time for students to respond to the teaching
5. Teaching according to student's ability

Five items were developed in Chinese to measure the above characteristics in this study. An example of the item is "Teaching is according to my ability".

Teacher-student relationship

The TSR is defined as interactions characterized by warmth, closeness, support, friendliness and caring between teachers and individual students (Emslander et al., 2023).

Four items were developed in Chinese to measure the above characteristics in this study. An example is "Student perceives the teacher as friendly".

Student engagement

Fredricks et al. (2004) proposed that StEn is a multi-dimensional construct which includes behavioral engagement, emotional engagement and cognitive engagement. Behavioral engagement is an important dimension of StEn, which includes effort, persistence, attention, participation, following rules and the absence of disruptive behaviors. With the review of literature, StEn is measured as one concept and operationally defined in this study that students actively engage in classroom study with good atmosphere, smooth running of activities, no improper behavior, following rules with active interaction. Five items were developed to measure the above characteristics in Chinese. An example of the items is "There is good atmosphere of learning in the classroom."

Student achievement

After asking students to respond to the questionnaire, their scores in final examination of mathematics were recorded after a few months and are used as a measure of student's achievement in mathematics in this research.

RESULTS

All 98 students from form 4 filled out the questionnaire. After the procedure for checking and cleaning the data, it was found that the 98 completed questionnaires could be used for conducting confirmatory factor analysis (CFA) by using AMOS version 28.

Demographic Data of the Sample

There are 63 male students and 35 female students responding to the questionnaire.

Descriptive Statistics of Responses to Each Item of the Questionnaire and the Examination Score

A descriptive analysis of the responses to each item of the questionnaire shows that the mean score of each item ranges from 3.39 to 4.05 out of a maximum of 5, and the standard deviation of each score ranges from 0.78 to 0.98. The skewness of all responses lies between -1 to 1. The kurtosis of all responses except two also lies between -1 to 1. The kurtosis of the two responses are 1.2 and 1.5,

Table 1. Composite reliability, Cronbach's alpha reliability, average variance extracted, and correlations among factors

Factor	Composite reliability	Cronbach's alpha reliability	Average variance extracted	R	T	M
Teacher-student relationship (R)	0.893	0.871	0.628	1		
Teaching effectiveness (T)	0.939	0.938	0.755	0.854*	1	
Student engagement (E)	0.885	0.893	0.608	0.741*	0.724*	1

Note. *p < 0.001

Table 2. Descriptive statistics of the three factors and achievement score, and the difference with the theoretical means of the three factors

Variable	Mean	Standard deviation	Skewness	Kurtosis	df	Mean difference with 3	t-value
Effective teaching	3.66	0.729	-0.732	0.968	97	0.638	8.168*
Teacher-student relationship	3.63	0.716	-0.542	1.090	97	0.608	8.052*
Engagement	3.67	0.653	-0.479	1.080	97	0.681	10.227*
Achievement	34.28	19.26	0.383	0.485	-	-	-

Note. *p < 0.001

respectively. From the above analyses, normal distribution of all responses may be assumed. The following parametric analyses can further proceed.

Confirmatory Factor Analysis

The measurement scales for each of the three constructs, TE, TSR, and StEn were analyzed by CFA using Amos version 28 as three distinct constructs. The result of analyses supports three distinct factors with an excellent fit (CFI: 0.959; IFI:0.946 RMSEA: 0.082). **Table 1** reports the reliability, average variance extracted and correlations among factors which show that the three constructs are reliable, with composite reliability ranging from 0.885 to 0.939, and Cronbach's alpha from 0.871 to 0.938. Since all the variances extracted are bigger than 0.5 and the correlations among factors by CFA are below 0.852, the results support that the three constructs have discriminant validity and reliability.

Since the three constructs are developed with reference to literature and inspected by experts in the field, they should have content validity. The structural equation model analyzed below can also support that the three constructs have convergent validity (**Table 1**).

Descriptive Statistics of and Correlations Among the Three Factors and Achievement Score

The descriptive statistics and the difference between the theoretical means of the three factors and achievement scores are reported in **Table 2**. The results show that the four variables do not deviate from normal distribution. Students, in general, have positive views on TE, the relationship with teachers and their engagement in learning as all the variables are significantly higher than 3 out of a Likert scale of 5.

Structural Equation Modelling

The above analyses show that there are significant relationships among various factors. To test the validity of the 4 hypotheses, structural equation models are needed. Model 1, according to the theoretical model, is put forward according to analysis by AMOS version 28. The examination scores are the results of the final examination a few months after students respond to the questionnaire. The goodness of fit indices of the structural equation model shows that the data have excellent fit indices with the theoretical framework (CFI=0.970; IFI=0.971; TLI=0.960; RMSEA= 0.069).

However, it is possible that the actual causal relationships among the three factors may not be as proposed in the theoretical model. For example, it may be possible that the TSR is the most important factor which will affect a teacher's TE and StEn instead of TE as the predicting variable of other variables. In order to verify whether the proposed theoretical framework is the most appropriate model describing the relationships among various variables, several competing models as shown in **Figure 2** and structural equation modelling analyses were performed for each model to see whether the data support the proposed theoretical model of this study. Model 1 is the model according to the theoretical framework. Model 2 and model 3 are different models with TE as the predicting variable but with different paths to other variables. Model 4 and model 5 are two models with the TSR as the predicting variable with different paths to other variables. The goodness of fit indices of different models are shown in **Table 3**.

Table 3 shows that model 1 has the smallest chi-square as compared with model 2 to model 4 which have the same degree of freedom, it is a better model as compared to them. Model 5 has one degree of freedom less than model 1 but has an increase of 7 in chi-square, implying that model 1 is a much better model as compared to model 5. Hence, model 1 represents the best model as compared to all other four models by considering the small changes in chi-square. If we compare the models by all other fit indices, it can be seen that model 1 is consistently better than all other models. Hence, it can be concluded that model 1 is the best model according to analysis of data.

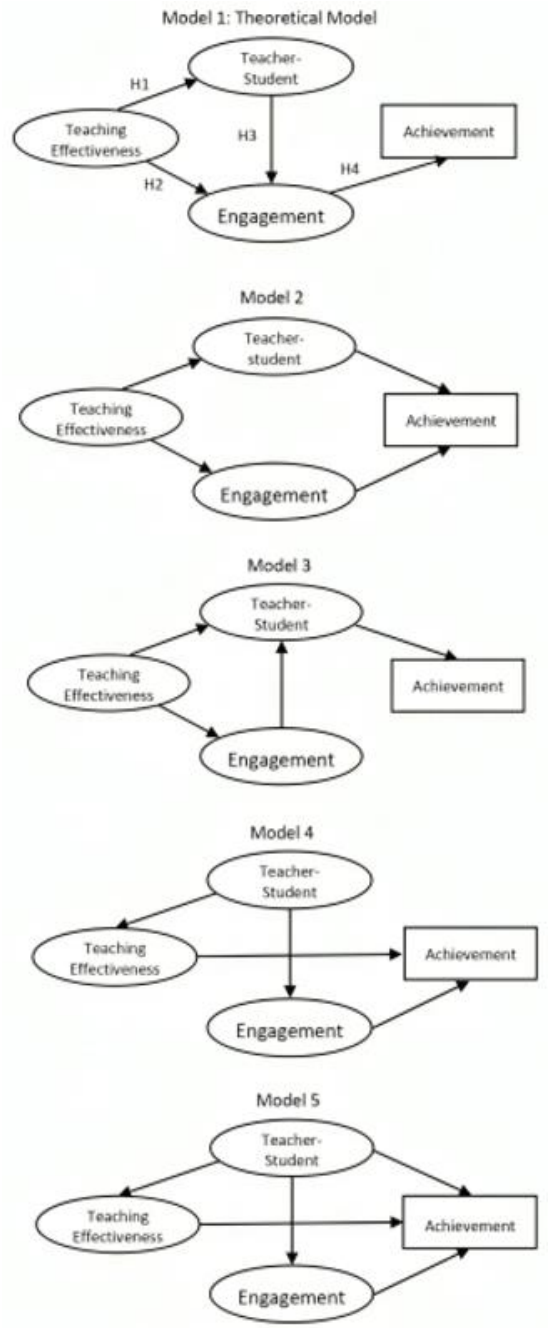

Figure 2. Competing models (Source: Authors' own elaboration)

Table 3. Model comparison

Model	CS	df	CS/df	CFI	RMSEA
1 (TM)	114.112	78	1.463	0.970	0.069
2	133.234	78	1.708	0.954	0.085
3	116.139	78	1.489	0.968	0.071
4	121.695	78	1.560	0.964	0.076
5	121.157	77	1.573	0.963	0.077

Note. TM: Theoretical model & CS: Chi-square

Relationships among Variables

Since model 1 is best fitted by the data among various competing models, this study confirms the proposed theoretical model according to the data obtained. It is used to interpret the relationships among various variables in this study. The results of the model with standardized direct effects and total effects of one variable on the other are reported in **Table 4**.

Table 4. Standardized direct and total effects among various variables

Relation between variables	SDE	STE
TE to TSR	0.869*	0.869*
TSR to StEn	0.550*	0.550*
TE to StEn	0.450*	0.929*
StEn to achievement	0.380*	0.380*
TE to achievement	-	0.353*

Note. *p < 0.001; SDE: Standardized direct effect; STE: Standardized total effect

DISCUSSION

The proposed theoretical model suggests that TE has a positive direct effect on TSR and StEn. Furthermore, the TSR has a significant relationship with StEn which in turn affects student achievement. The relationships are proposed and supported by related research studies. The analyses above show that model 1 is the best model fitting the data compared to other models. Furthermore, the goodness of fit indices shows that the data obtained provide excellent fit indices for model 1. Hence the findings of this study confirm the significant relationships among various variables in this study. Since the exam scores were taken a few months after measuring various factors by questionnaire, there is evidence to support the causal relationship of the three factors measured on student achievement. However, the data measuring the three variables are taken by students' responses to the questionnaire simultaneously, and the causal relationships among them cannot be confirmed. The results from the structural equation model only suggest that the causal relationship aligns with the proposed theoretical model. An interpretation of the results is put forward, as follows.

Effects of Teaching Effectiveness on Teacher-Student Relationship

This study finds that TE has significant standardized direct effects of 0.869 on TSR. The result supports hypothesis 1, which states that effective teaching has positive effects on the TSR.

The relationship can be interpreted, as follows. Relationship between people has to be established through knowing each other, social interaction, working together, mutual understanding and affiliation need. The social interaction between teachers and students is mainly through the teaching and learning process. Some students may only know the teacher after the first lesson. Effective teaching requires social interaction between teacher and students and mutual understanding. Effective teaching can fulfil the achievement and affiliation needs of students. Form 4 students in Hong

Kong usually attach great importance to academic achievement because their future well-being is perceived to be seriously affected by their examination results. If a teacher can teach effectively, students' achievement and affiliation needs can be satisfied. The teacher will be well-received by students. Hence, a better relationship between teacher and students will follow.

Effects of Teacher-Student Relationship on Student Engagement

This study finds that the TSR has a significant standardized direct effect of 0.550 on StEn. The result supports hypothesis 3 which states that TSR has positive effects on StEn.

The effect can be explained by social learning theory and the affiliation needs of students. If the relationship between teacher and students is good, students' affiliation needs will be met, and students will take more action to further enhance their affiliation needs. They are more willing to have social interaction with the teacher in the classroom, follow their teacher's teaching, positively respond to the teacher's request to complete learning tasks and maintain a good atmosphere of learning. Hence, good TSR can lead to students' engagement in learning.

Effects of Teaching Effectiveness on Student Engagement

This study finds that TE has significant direct effects and a total effect on StEn of 0.450 and 0.929, respectively. The result supports hypothesis 2 which states that Effective teaching has positive effects on StEn.

With effective teaching delivered by a teacher, students' understanding of the concept taught and students' ability to solve mathematical problems in the classroom will be enhanced. They will be motivated to actively engage in learning because their achievement need can be satisfied. Students' engagement in learning will be further reinforced after they find more satisfaction in completing tasks given by teachers. Hence TE can have a direct effect on StEn.

TE can also affect TSRs which in turn affect StEn. Hence, in addition to the above direct effect, TE can affect students' engagement through teacher-students relationships as a intervening variable.

Effects of Student Engagement on Students' Examination Scores in Mathematics

This study finds that StEn has a significant standardized direct effect of 0.380 on StEn. The result supports hypothesis 4 which states that StEn has positive effects on student achievement. The relationship can explain a 14.5% variance in students' achievement in the examinations. Since there are many factors affecting the achievement of students, a single factor of engagement explaining around fourteen per cent of the variance is quite significant.

The result of this study supports research studies showing that there is a significant relationship between StEn and academic achievement (Gunuc, 2014; Hughes et al., 2008). StEn is very important for students to learn mathematics. A student needs to solve mathematical problems in order to have a good examination score. In the learning of mathematics, students are often given the task of solving a mathematical problem. Without actively taking part in solving a problem in the classroom, students may not be able to solve a problem in examination. Hence, engagement is a very important factor for enhancing achievement in mathematics.

CONCLUSION

The study proposes a theoretical model of the following causal relationships: effective teaching has a positive significant effect on the TSR and StEn, the TSR has a positive significant effect on StEn, and StEn has a positive effect on student achievement. The data and analysis support the relationship of the theoretical model and all four hypotheses.

There are many studies supporting the idea that TE, TSR and StEn have positive effects on student achievement. However, the exact causal relationships among the variables are difficult to identify with support from data. This study tries to propose a set of causal relationships among the variables and structural equation modelling analysis seems to confirm the causal relationships among various variables according to the theoretical model. The findings may cast light on the casual relationships among various variables. However, further research has to be conducted to have a better understanding of the relationships.

The results of this study suggest that TE is an important predictor of TSR, StEn, and student achievement in mathematics. TE has always been treated as a very important aspect leading to school effectiveness and students' learning effectiveness. There are various dimensions of TE, but the effective teaching strategy in this study can explain around 14% of the variance of student achievement in mathematics, which is quite significant. Numerous factors are found to be related to students' achievement such as intelligence and other personal factors of student, family background, motivation, environment, school factors, and social factors. In this study, just one dimension of TE, teaching strategy, is found to have a total effect on 14% of the variance of students' achievement in mathematics. Teachers and the school administration should attach great importance to paying more effort to improve the teaching strategy adopted by teachers.

StEn is found to have a direct causal effect on students' mathematical achievement. It is an intervening variable between TE and student achievement. StEn can

be relatively easy to observe. The observation can be used to estimate the future achievement of students. If StEn is not up to expectation, a school or teacher can take some remedial measures to improve students' engagement to enhance students' achievement.

The TSR is found to have a direct causal effect on students' engagement, which in turn affects mathematical achievement. It is an intervening variable between TE and student achievement. The TSR can also be easily felt by teachers and students. The observation of the TSR can be used to estimate the future achievement of the students. If a TSR is not up to expectation, a school or teacher can take some remedial measures to improve the relationship to enhance students' achievement.

Limitations

This study conducts quantitative research in a school which supports a proposed theoretical model for TE, TSR, StEn and mathematical achievement in a school. Only one dimension of TE, instructional effectiveness, was measured in this study. Further studies including all dimensions of TE (Stronge et al., 2011) may cast light on the effects of various dimensions of TE on TSR, StEn and StEn.

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Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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